CBSEASSISTANCE.COM

CHAPTER 3
THE WORLD OF NUMBERS
CLASS 9
QUESTION BANK BASED ON NCERT

Comprehensive Question Bank: The World of Numbers
CBSE Grade 9 Mathematics (NEP 2020 Guidelines)

SECTION A: 1-Mark Questions (40 Questions)

Multiple Choice Questions (Q1 - Q20)

(Hints provided for all questions.)

1. What fundamental human necessity led to the birth of Natural Numbers?
A) The desire to build cities
B) The study of the stars
C) The need to keep count of assets like cattle
D) The formulation of laws
Hint: Think about what early agricultural societies needed most practically on a daily basis.

2. The Lebombo Bone, which is one of the earliest physical evidences of humanity tracking numbers, features how many distinct notches?
A) 20
B) 29
C) 30
D) 35
Hint: It's associated with a lunar phase counter, closely matching the days of a lunar month.

3. Which set of numbers is demonstrated in one of the columns on the Ishango Bone?
A) Prime numbers between 10 and 20
B) Powers of 10
C) Multiples of 2
D) Fibonacci sequence
Hint: The numbers 11, 13, 17, and 19 share a specific mathematical property regarding divisibility.

4. In the *Lalitavistara* (4th century BCE), Buddha describes names for numbers up to 10^{53}. What is this number called?
A) Parārdha
B) Śhūnyatā
C) Bindu

D) Tallakṣhaṇa
Hint: Look closely at the section discussing the Indian context of Trade and Astronomy.

5. According to Brahmagupta's rules, what is the result of a "debt plus a debt"?
A) A fortune
B) Zero
C) A debt
D) An irrational number
Hint: If you borrow money from a friend and then borrow more money the next day, what is your overall financial state?

6. The symbol for the set of Integers is $\mathbb{Z}$. Which language does the word *Zahlen*, from which this symbol is derived, belong to?
A) Greek
B) Latin
C) German
D) Sanskrit
Hint: Look at the section expanding the horizon of integers; it's a major European language.

7. A rational number is defined as any number that can be expressed in the form $\frac{p}{q}$. What is the essential condition for q?
A) $q = 0$
B) $q \neq 0$
C) $q = 1$
D) $q = p$
Hint: Division by zero breaks the rules of mathematics and is undefined.

8. According to the density property of rational numbers, how many rational numbers exist between the integers 1 and 2?
A) 1
B) 10
C) 100
D) Infinitely many
Hint: No matter how close two rational numbers are, you can always find another one by taking their average.

9. If the rational number $x = \frac{p}{q}$ has a terminating decimal expansion, then the prime factorization of q must contain only which prime numbers?
A) 2 and 3
B) 3 and 5
C) 2 and 5
D) 7 and 11
Hint: Our decimal system is base-10. What are the prime building blocks of 10?

10. What physical symbol was used in the *Bakhśhālī Manuscript* to represent zero?
A) An empty circle

B) A bold dot (bindu)
C) A placeholder tally
D) An ’X’ mark
Hint: Look for the Sanskrit term mentioned before Brahmagupta gave rules to zero.

11. The absolute value of a rational number x is defined as $|x|$. Which of the following is true for $\left|-\frac{5}{3}\right|$?
A) $-\frac{5}{3}$
B) 0
C) $\frac{5}{3}$
D) $\frac{3}{5}$
Hint: Absolute value represents the physical distance from 0 on the number line, which is always non-negative.

12. Which ancient Indian text composed around 800 BCE encountered lengths that defied fractions, leading to early encounters with irrational numbers?
A) Rigveda
B) Yoga Sutras
C) Śhulbasūtra
D) Upanishads
Hint: This text was a manual for constructing complex geometric fire altars.

13. A merchant takes a loan of 850, representing a debt. In Brahmagupta’s terminology, this negative number is called: [**Competency-Based**]
A) Dhana
B) Ṛiṇa
C) Vṛttis
D) Asanna
Hint: Brahmagupta used ’Dhana’ for Fortunes. What is the opposite term he used?

14. What happens when the repeating decimal $0.\overline{142857}$ is multiplied by 3?
A) The digits become completely random.
B) It results in a terminating decimal.
C) The same digits shift in a cyclic circle to become $0.\overline{428571}$.
D) The sequence becomes $0.\overline{857142}$.
Hint: 142857 *is a “cyclic number”. The digits don’t change entirely, they just rotate.*

15. Who first proved the irrationality of $\sqrt{2}$ using proof by contradiction?
A) Aryabhata
B) Madhava of Sangamagrama
C) Hippasus
D) Brahmagupta
Hint: He was a member of the Pythagorean school around 400 BCE.

16. Which mathematician formally transformed the philosophical void into an operational mathematical number (Zero) in 628 CE?

A) Aryabhata
B) Patanjali
C) Brahmagupta
D) Lambert
Hint: He wrote the Brahmasphuṭasiddhānta and established arithmetic laws for fortunes and debts.

17. The union of Rational Numbers and Irrational Numbers creates which continuous, unbroken set?
A) Natural Numbers
B) Real Numbers
C) Imaginary Numbers
D) Integers
Hint: Together, they make up all possible lengths and physical measurements on the number line.

18. To convert the general repeating decimal $0.1\overline{6}$ into a fraction, what is the first power of 10 you should multiply by to shift the non-repeating part?
A) 10^1
B) 10^2
C) 10^3
D) 10^4
Hint: You need to shift the decimal past the single non-repeating digit '1'.

19. According to Aryabhata, the fractional approximation $\frac{3927}{1250}$ for π was called an *asanna*. What does *asanna* mean?
A) Exact value
B) Approximation
C) Infinite series
D) Cyclic number
Hint: Since π is irrational, a fraction can only ever be "close" to the real value, not an exact match.

20. If a number cannot exist on the Real Number line (like the square root of -1), what type of number is it called?
A) Negative number
B) Rational Number
C) Imaginary Number
D) Irrational Number
Hint: Stepping "completely off the line" requires a new dimension of numbers, denoted by 'i'.

Objective Type Questions (Q21 - Q30)

Fill in the blanks (Q21 - Q25)

21. The simple act of matching one object to another without written symbols is called correspondence.
Hint: Think of the ancient herder matching exactly one cow to one pebble.

22. In the Indian philosophical tradition, the concept of meaning emptiness or nothingness, paved the way for the mathematical zero.
Hint: This is the Sanskrit term found in ancient Buddhist literature and Upanishads.

23. A positive number representing wealth or assets was referred to by Brahmagupta as .
Hint: Look for the Sanskrit term used by Brahmagupta as the opposite of Ṛiṇa.

24. To find a rational number between two rational numbers a and b, you can take their , given by the formula $\frac{a+b}{2}$.
Hint: This mathematical operation finds the exact middle point between two values.

25. An exact mathematical formula for π cannot be a single fraction; as discovered by Mādhava, it requires a sum.
Hint: Madhava realised a single fraction wasn't enough; the series must go on without ending.

True or False (Q26 - Q30)

26. The decimal expansion of an irrational number is always non-terminating and repeating.
Hint: If a decimal repeats, it can be written as a fraction $\frac{p}{q}$. Is that true for irrational numbers?

27. Rational numbers are closed under division, provided that one does not divide by zero.
Hint: Dividing any rational number by another non-zero rational number always yields a rational number.

28. Brahmagupta's rule states that the product of a debt and a fortune is a fortune.
Hint: Think of $(-3) \times (+4)$. What is the mathematical sign of the answer?

29. The number $0.\overline{9}$ is exactly equal to 1.
Hint: Let $x = 0.\overline{9}$, multiply by 10 and subtract x. Check what value you get for x.

30. Two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal if and only if $ad = bc$.
Hint: Try cross-multiplying equivalent fractions like $\frac{1}{2}$ and $\frac{2}{4}$.

Assertion-Reasoning Type Questions (Q31 - Q40)

Options for Q31-Q40:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

31. **Assertion (A):** The sequence of numbers 11, 13, 17, 19 on the Ishango bone represents prime numbers.
Reason (R): Prime numbers are numbers that have no divisors other than 1 and themselves.
Hint: Check if 11, 13, 17, and 19 have any other divisors.

32. **Assertion (A):** Brahmagupta formally introduced negative numbers by placing them to the left of zero on the number line.
Reason (R): Brahmagupta grounded his mathematics in the reality of commerce, using "Debts" to represent negative numbers.
Hint: Did Brahmagupta formally introduce them? Yes. Did he use commerce terminology? Yes. Are these facts linked?

33. **Assertion (A):** The rational number $\frac{3}{20}$ will have a terminating decimal expansion.
Reason (R): The denominator 20 has prime factors other than 2 and 5.
Hint: Factorize 20 into $2^2 \times 5$. *Do you see any prime factors other than 2 or 5?*

34. **Assertion (A):** There are infinitely many rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.
Reason (R): The set of integers is infinitely dense.
Hint: Rational numbers are densely packed (you can keep taking averages), but are integers dense? Are there integers between 1 and 2?

35. **Assertion (A):** $|-10|$ is equal to 10.
Reason (R): The absolute value of a rational number represents its distance from 0 on the number line and is always non-negative.
Hint: Absolute value measures physical distance, which cannot be negative.

36. **Assertion (A):** $\sqrt{2}$ can be written exactly in the form $\frac{p}{q}$, where p and q are integers.
Reason (R): Numbers on the number line that cannot be expressed as a ratio of integers are called Irrational Numbers.
Hint: Is $\sqrt{2}$ *rational or irrational? Can it be written as* $\frac{p}{q}$*?*

37. **Assertion (A):** When proving $\sqrt{2}$ is irrational, assuming $\sqrt{2} = \frac{p}{q}$ (co-prime) leads to the deduction that both p and q are even.
Reason (R): If the square of a number is even, the number itself must be even.
Hint: Remember the logic steps for proof by contradiction. If p^2 *is a multiple of 2, what about* p*?*

38. **Assertion (A):** According to Brahmagupta, when zero is added to a number a, the result is 0.
Reason (R): Brahmagupta stated that $a + 0 = a$.
Hint: If you add nothing to your wealth, do you lose everything or keep what you have?

39. **Assertion (A):** The decimal $0.\overline{142857}$ is a rational number.
Reason (R): Any number with a repeating decimal block can be expressed as a fraction of integers.
Hint: Only irrational numbers have non-terminating, NON-repeating decimals.

40. **Assertion (A):** Mādhava of Sangamagrama used a single finite fraction to represent the exact value of π.
Reason (R): Because π is irrational, no finitely many fractions can ever provide a perfect formula for it.
Hint: Did Madhava use a finite fraction or an infinite sum?

SECTION B: 2-Mark Questions (15 Questions)

41. Using Brahmagupta's rules for commerce, evaluate the following: "A debt of 8 plus a debt of 6". Express your answer mathematically.
Hint: Translate the words into addition of two negative numbers.

42. State the absolute value of $-\frac{19}{7}$ and explain what it represents on the number line.
Hint: The absolute value $|x|$ removes the negative sign to show physical magnitude/distance.

43. Using the formula $\frac{a+b}{2}$, find exactly one rational number between $\frac{1}{4}$ and $\frac{1}{2}$.
Hint: Add the two fractions using a common denominator, then divide the result by 2.

44. The temperature in Ladakh is $5°C$ at noon. By midnight, it drops by $12°C$. Represent this using integers and find the midnight temperature. [**Competency-Based**]
Hint: Start at $+5$, then add (-12).

45. Determine whether the rational number $\frac{7}{20}$ will have a terminating or non-terminating decimal expansion without performing long division. Justify your answer.
Hint: Look closely at the prime factorization of the denominator 20 ($2^2 \times 5$).

46. Simplify the expression using the distributive property: $\frac{1}{2} \times \left(\frac{3}{4} + \frac{5}{4}\right)$.
Hint: Add the fractions inside the bracket first (they share a denominator), then multiply.

47. What is the result when you multiply a "debt" of -3 by a "debt" of -4 according to Brahmagupta? Why is the result positive?
Hint: Think of multiplying by a negative as "taking away". Removing a debt effectively makes you richer.

48. Convert the terminating decimal 2.1625 into a rational number in the simplest form $\frac{p}{q}$.
Hint: Write it as 21625 divided by 10000, then simplify the fraction.

49. Evaluate $|-5| - |3|$.
Hint: Calculate the absolute value $|-5|$ first, then subtract $|3|$.

50. Prove that the rational numbers $\frac{2}{3}$ and $\frac{1013}{1519.5}$ are equivalent.
Hint: Cross-multiply ($ad = bc$) to check if both products yield the same number.

51. A merchant exchanges 15 ingots for every 2 bags of spices. If he brings 12 bags, how many ingots will he receive? [**Competency-Based**]
Hint: Find out how many "sets" of 2 bags are in 12 bags, then multiply by 15.

52. Explain briefly what the cyclic property of the number $\frac{1}{7}$ ($0.\overline{142857}$) means.
Hint: Watch what happens to the sequence 142857 when you multiply it by 2, 3, 4, etc.

53. Identify if $23.5601856122398747901 20...$ (with no repeating block) is rational or irrational, and state why.
Hint: Does the decimal terminate? Does it have a repeating cycle?

54. Write the sequence of the first four positive powers of 10 as pondered by ancient Indian philosophers during the Vedic times.
Hint: Start with 10^1 and continue up to 10^4.

55. Evaluate the division of two rational numbers: $\left(-\frac{20}{7}\right) \div \frac{4}{14}$.
Hint: Use the "Keep, Change, Flip" rule—multiply the first fraction by the reciprocal of the second.

SECTION C: 3-Mark Questions (15 Questions)

56. Evaluate using Brahmagupta's laws:
(i) $(-12) \times 5$
(ii) $0 - (-14)$
(iii) $(-20) \div 4$
Hint: Apply integer multiplication (debt x fortune), subtraction, and division rules.

57. A spice trader takes a loan (debt) of 850. The next day, he makes a profit (fortune) of 1,200. The following week, he incurs a loss of 450. Write this sequence as an equation using integers and calculate his final financial standing. [**Competency-Based**]
Hint: Set up the equation: $-850 + 1200 - 450$, and solve sequentially.

58. Prove that the pure repeating decimal $0.\overline{45}$ is equal to $\frac{5}{11}$.
Hint: Let $x = 0.4545...$, multiply both sides by 100, and subtract the original equation.

59. Convert the general repeating decimal $0.1\overline{6}$ into the form $\frac{p}{q}$ using algebraic steps.
Hint: Multiply by 10 to shift the '1', then multiply by 10 again to shift the '6'. Subtract the two resulting equations.

60. Find three distinct rational numbers that lie strictly between $-\frac{1}{2}$ and $\frac{1}{4}$.
Hint: Convert both fractions to a common denominator (like 8) to easily spot integer numerators between them.

61. A tailor has $15\frac{3}{4}$ metres of fine silk. If making one kurta requires $2\frac{1}{4}$ metres of silk, exactly how many kurtas can he make? [**Competency-Based**]
Hint: Convert the mixed numbers into improper fractions ($\frac{63}{4}$ and $\frac{9}{4}$) and divide.

62. Find the rational number x such that: $\frac{5}{6}\left(x + \frac{3}{5}\right) = \frac{5}{6}x + \frac{1}{2}$.
Hint: Distribute the $\frac{5}{6}$ into the bracket on the left side and see what happens to the equation.

63. Convert $2.3\overline{57}$ into a rational fraction $\frac{p}{q}$.
Hint: Multiply by 10 to move the non-repeating '3' to the left, then by 1000 to move the full repeating cycle.

64. Using a number line, visually represent the rational numbers $\frac{3}{4}, \frac{5}{4}$, and $\frac{-3}{4}$.
Hint: Draw a number line and divide the distance between every integer unit into 4 equal segments.

65. Without performing division, determine whether $\frac{18}{125}$ is terminating or non-terminating. If it terminates, state how many decimal places it will have and evaluate the decimal.
Hint: Prime factorize 125 (5^3). Multiply the numerator and denominator by 2^3 to turn the bottom into 1000.

66. Show algebraically that $0.\overline{9}$ equals 1. Explain this conceptually regarding the non-uniqueness of decimal representations.
Hint: Let $x = 0.999\ldots$, multiply by 10, subtract x. Terminating decimals can be written with trailing 9s.

67. Distinguish clearly between the Bakhśhālī Manuscript's contribution to Zero and Brahmagupta's contribution.
Hint: One provided a physical dot/placeholder (bindu), while the other provided actual arithmetic laws ($a - a = 0$).

68. Let $a = \frac{7}{12}$ and $b = \frac{5}{6}$. Calculate the sum $(a + b)$ and the difference $(b - a)$.
Hint: Find a common denominator (12) before adding or subtracting.

69. Three rational numbers x, y, z satisfy $x + y + z = 0$ and $xy + yz + zx = 0$. Show that all the rational numbers x, y, z must be simultaneously zero.
Hint: Use the algebraic identity for $(x + y + z)^2$. If the sum of squares is 0, what must be true for x, y, and z?

70. Explain the concept of Śhūnyatā as used in ancient Buddhist literature and how it conceptually permitted the mathematical acceptance of zero in India.
Hint: Emptiness was a profound state in meditation, which allowed Indian philosophers to easily accept 'nothingness' mathematically.

SECTION D: 5-Mark Questions (10 Questions)

71. Step-by-step, explain Hippasus's proof by contradiction that $\sqrt{2}$ is an irrational number.
Hint: Assume $\sqrt{2} = \frac{p}{q}$ (co-prime). Square both sides, show p^2 is even, meaning p is even. Substitute to show q is also even.

72. Extend the logic of Hippasus's proof to prove that $\sqrt{3}$ is an irrational number. State the assumption clearly, deduce the contradiction involving the divisibility by 3.
Hint: Follow the exact same logic as the $\sqrt{2}$ proof, but show p^2 is a multiple of 3, meaning p is a multiple of 3, and so is q.

73. Describe the exact geometric steps required to construct a line segment of length $\sqrt{2}$ and how to mark its position on the real number line, starting from the origin. [**Competency-Based**]
Hint: Create a right-angled triangle on the number line with base 1 and height 1. Use Pythagoras and a compass to draw an arc down.

74. Show that the rational number $\frac{a+b}{2}$ always lies strictly between the rational numbers a and b (assume $a < b$). Verify this by finding a rational number between $\frac{2}{3}$ and $\frac{3}{5}$.
Hint: Manipulate the inequality $a < b$ by adding 'a' to both sides, and then separately adding 'b' to both sides. Find a common denominator to verify.

75. Convert the decimal $2.45\overline{37}$ into the form $\frac{p}{q}$. Show all the algebraic shifting of the decimal point using powers of 10.
Hint: Let $x = 2.453737\ldots$ First, multiply by 100 to shift the "45", then multiply the original x by 10000 to shift one full "37" cycle.

76. Let $a = \frac{7}{12}$ and $b = \frac{5}{6}$. Express both a and b in the form $\frac{k_1}{m}$ and $\frac{k_2}{m}$ such that you can write exactly five distinct rational numbers lying between a and b keeping an integer numerator. Show your complete working.
Hint: Between $\frac{7}{12}$ and $\frac{10}{12}$ there are only 2 integers. Multiply the numerator and denominator by a large enough factor (like 6) to widen the gap between numerators.

77. Perform the long division for $\frac{1}{7}$. Identify the repeating block. Then, without doing long division, evaluate the decimal expansion for $\frac{3}{7}$ and $\frac{5}{7}$ utilizing the cyclic property of the digits.
Hint: Find the repeating block 142857. For 3/7, start the cycle at the 3rd lowest digit. For 5/7, start at the 5th lowest digit.

78. Trace the evolution of our world of numbers from Natural Numbers to Real Numbers. Define $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{I}$, and $\mathbb{R}$, explicitly mentioning how each set resolves a limitation of the previous one (e.g., debts, parts of wholes, incommensurable lengths). [**Competency-Based**]
Hint: Tell the story: Counting objects → Tracking debts/zero → Dividing fields/recipes → Measuring diagonals of squares → Uniting them all.

79. A rational number has a terminating decimal expansion whose last non-zero digit occurs in the 4th decimal place. Show that such a number can be written in the form $\frac{p}{10^4}$ where p is an integer not divisible by 10. Is it necessary that the denominator of this rational number, when written in its lowest form, is divisible by 2^4 or 5^4? Give reasons.
Hint: Since p is not divisible by 10, it cannot contain BOTH a factor of 2 AND a factor of 5. Think about what happens when you reduce the fraction.

80. Mādhava of Sangamagrama expressed π using an infinite sum: $\pi = 4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$. Explain why a finite fraction cannot perfectly represent π, referring to its irrationality. Calculate the approximation of π by summing just the first 4 terms of Madhava's series inside the bracket.
Hint: Irrational decimals never end or loop, so no single fraction p/q fits. Calculate $4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\right)$ *with a common denominator.*

SECTION E: Case Study Based Questions (Q81 - Q85)

(Strictly formatted as per CBSE guidelines: 1 Mark + 1 Mark + 2 Marks with internal choice).

81. **Case Study 1: The Ishango Bone**
Discovered near the headwaters of the Nile River dating to around 20,000 BCE, the Ishango bone is a mathematical marvel. One column on this bone groups asymmetrical notches into tallies of 11, 13, 17, and 19. Another column demonstrates the concept of multiplication by 2 (doubling). These artifacts show that the abstract concept of a 'number' is tens of thousands of years old.

   a. What special mathematical property is shared by the specific sequence 11, 13, 17, and 19? [**1 Mark**]

   b. The concept of tracking numbers on ancient bones originally relied on matching exactly one object to one tally. What is this concept called? [**1 Mark**]

   c. If the pattern of the sequence in the first column were to continue, what would be the next two numbers?

   OR

   If a prehistoric herder used the doubling column to double a count of 17 cattle, express the calculation and the final number of notches. [**2 Marks**]

Hint: Think about prime numbers for the sequence.

82. **Case Study 2: Brahmagupta's Ledger**
To answer what happens when you subtract a larger number from a smaller one, Brahmagupta grounded his mathematics in the reality of commerce. He recognised two states: Fortunes (*Dhana*), representing positive numbers/assets, and Debts (*Ṛiṇa*), representing negative numbers/debts. He laid down explicit arithmetic rules for these over 1,300 years ago.

   a. A merchant calculates: "A fortune of 7 minus zero". According to Brahmagupta's rules, what is the result? [**1 Mark**]

   b. By moving to the left of zero to track Debts, Brahmagupta formally created which mathematical set of numbers? State its name and symbol. [**1 Mark**]

c. A merchant has a debt of 15 and borrows 10 more. Express this mathematically using integers and find the final result.

OR

If a person takes on 5 debts of 4 each, what is the mathematical expression for this product, and what is the final state (fortune or debt)? [**2 Marks**]

Hint: Remember negative numbers represent Ṛiṇa (Debts).

83. **Case Study 3: The Search for Pi (π)**
For centuries, mathematicians sought fractional approximations for the irrational number π. Āryabhaṭa (499 CE) gave the highly accurate fraction $\frac{3927}{1250}$ but noted it was only an *asanna* (approximation). Later, Mādhava realised that to express an irrational number exactly, you cannot use a single fraction; you must use an infinite series.

a. Why can the exact value of π never be written as a single fraction $\frac{p}{q}$? [**1 Mark**]

b. What is the mathematical term for numbers whose decimal expansions never end and never repeat? [**1 Mark**]

c. Without doing long division, determine whether Āryabhaṭa's fraction $\frac{3927}{1250}$ will terminate or repeat. Justify using prime factorization.

OR

Write the first three terms of the infinite sum inside the bracket of Mādhava's formula for π. [**2 Marks**]

Hint: Look at the prime factors of 1250.

84. **Case Study 4: Cyclic Decimals**
Some rational numbers have repeating decimal representations. When we calculate the decimal for $\frac{1}{7}$, we get $0.\overline{142857}$. This repeating block is a cyclic number. The same digits simply shift in a cyclic circle when multiplied by 1 through 6.

a. When evaluating the long division for $\frac{1}{7}$, what are the possible remainders at each step? [**1 Mark**]

b. Why must the division of $\frac{1}{7}$ eventually create a repeating loop instead of continuing randomly? [**1 Mark**]

c. Based on the cyclic property, if $\frac{1}{7} = 0.\overline{142857}$, find the exact decimal value of $\frac{2}{7}$.

OR

Let $x = 0.\overline{142857}$. Explain step-by-step by what power of 10 you must multiply x to shift exactly one full repeating cycle to the left of the decimal point. [**2 Marks**]

Hint: The number of possible remainders dictates when the loop restarts.

85. **Case Study 5: The Revolution of Śhūnya**
For millennia, the number line started at 1. Civilisations like the Babylonians and Mayans used placeholders, but did not treat 'nothing' as a number to perform arithmetic on. Inspired by the ancient Indian philosophical concept of *Śhūnyatā* (emptiness) found in Buddhist literature and the Upanishads, Indian thinkers possessed the framework to welcome 'zeroness' into mathematics.

    a. In Patanjali's Yoga Sutras, what state is one trying to reach by emptying the mind of *vṛttis*? [**1 Mark**]

    b. Which ancient manuscript shows the physical transition of zero from a blank space to a bold dot (*bindu*)? [**1 Mark**]

    c. What is the fundamental difference between the Babylonian/Mayan placeholder and Brahmagupta's zero?

    OR

    State Brahmagupta's mathematical definition of zero using algebraic variables. [**2 Marks**]

Hint: Think about the difference between an empty space and an operational number.

EXPERT ANSWER KEY AND STEP-WISE MARKING SCHEME

SECTION A (1-Mark Questions)

(1 mark for each correct answer)

1. C
2. B
3. A
4. D
5. C
6. C
7. B
8. D
9. C

10. B
11. C
12. C
13. B
14. C
15. C
16. C
17. B
18. A
19. B
20. C
21. one-to-one
22. Śhūnyatā
23. Fortunes (Dhana)
24. average
25. infinite
26. False (Irrational decimals are non-terminating and *non-repeating*).
27. True
28. False (Product of debt and fortune is a debt; Product of two debts is a fortune).
29. True
30. True
31. (a)
32. (a)
33. (d) (Denominator 20 = $2^2 \times 5$. It ONLY has 2 and 5 as prime factors, so it WILL terminate. A is false).
34. (c) (Rational numbers are densely packed; Integers are NOT infinitely dense, they have strict gaps of 1 unit).
35. (a)

36. (d) ($\sqrt{2}$ is irrational, thus it cannot be expressed exactly as $\frac{p}{q}$. A is false).

37. (a)

38. (d) (When 0 is added to a, the result is a, not 0. A is false).

39. (a)

40. (d) (Madhava used an *infinite sum*, not a single finite fraction. A is false).

SECTION B (2-Mark Questions)

41. Expression: $(-8) + (-6)$ **[1 Mark]**

    Sum: $= -14$ **[1 Mark]**

42. $\left|-\frac{19}{7}\right| = \frac{19}{7}$ **[1 Mark]**

    It represents the physical, non-negative distance of $-\frac{19}{7}$ from 0 on the number line. **[1 Mark]**

43. Formula setup: $\frac{\frac{1}{4}+\frac{1}{2}}{2}$ **[0.5 Marks]**

    Finding common denominator: $\frac{\frac{3}{4}}{2}$ **[1 Mark]**

    Final Answer: $\frac{3}{8}$ **[0.5 Marks]**

44. Equation setup: $5 + (-12)$ **[1 Mark]**

    Midnight temperature: -7°C **[1 Mark]**

45. Prime factors of $20 = 2^2 \times 5$. **[1 Mark]**

    Since the prime factors consist exclusively of 2 and 5, the decimal expansion is terminating. **[1 Mark]**

46. Adding inside bracket: $\left(\frac{3+5}{4}\right) = \frac{8}{4} = 2$ **[1 Mark]**

    Multiplying: $\frac{1}{2} \times 2 = 1$ **[1 Mark]**

47. $(-3) \times (-4) = 12$ **[1 Mark]**

    According to Brahmagupta, the removal (subtraction/negative multiplication) of a debt is effectively a gain (fortune), making the result positive. **[1 Mark]**

48. Initial fraction: $\frac{21625}{10000}$ **[1 Mark]**

Completely simplified answer: $173/80$ **[1 Mark]**

49. $|-5| = 5$ and $|3| = 3$ **[1 Mark]**

$5 - 3 = 2$ **[1 Mark]**

50. Cross-multiply rule ($ad = bc$): $2 \times 1519.5 = 3039$ **[1 Mark]**

$3 \times 1013 = 3039$. Since products are equal, fractions are equivalent. **[1 Mark]**

51. Identifying sets: 12 bags is equal to $12 \div 2 = 6$ sets of 2 bags. **[1 Mark]**

Multiplying by ratio: $6 \times 15 = 90$ ingots. **[1 Mark]**

52. Definition: The digits 142857 do not change when multiplied by 1 through 6. **[1 Mark]**

They simply shift in a continuous circular loop. **[1 Mark]**

53. It is Irrational. **[1 Mark]**

Reason: Its decimal expansion is non-terminating and non-repeating. **[1 Mark]**

54. $10^1 = 10, 10^2 = 100, 10^3 = 1000, 10^4 = 10000$. **[0.5 Marks each = 2 Marks]**

55. Applying reciprocal: $-\frac{20}{7} \times \frac{14}{4}$ **[1 Mark]**

Simplifying: $-5 \times 2 = -10$ **[1 Mark]**

SECTION C (3-Mark Questions)

56. (i) $(-12) \times 5 = -60$ **[1 Mark]**

(ii) $0 - (-14) = +14$ **[1 Mark]**

(iii) $(-20) \div 4 = -5$ **[1 Mark]**

57. Identifying terms: Loan = -850, Profit = $+1200$, Loss = -450. **[1 Mark]**

Equation: $-850 + 1200 - 450$ **[1 Mark]**

Solving: $350 - 450 = -100$. Final standing is a debt of 100. **[1 Mark]**

58. Let $x = 0.4545...$ (Eq 1). Multiply by 100: $100x = 45.4545...$ (Eq 2). **[1 Mark]**

Subtract Eq 1 from Eq 2: $99x = 45$. **[1 Mark]**

$x = \frac{45}{99} = \frac{5}{11}$. **[1 Mark]**

59. Let $x = 0.1666...$ Multiply by 10: $10x = 1.666...$ (Eq 1). **[1 Mark]**

Multiply by 10 again: $100x = 16.666...$ (Eq 2). **[1 Mark]**

Subtract: $90x = 15 \Rightarrow x = \frac{15}{90} = \frac{1}{6}$. **[1 Mark]**

60. Convert to a common denominator (e.g., 8): $-\frac{4}{8}$ and $\frac{2}{8}$. **[1.5 Marks]**

Rational numbers between them: $-\frac{3}{8}, -\frac{2}{8}, \frac{1}{8}$. **[1.5 Marks]**

61. Improper fractions: Total silk = $\frac{63}{4}$ m. Requirement = $\frac{9}{4}$ m. **[1 Mark]**

Division setup: $\frac{63}{4} \div \frac{9}{4}$ **[1 Mark]**

Solving: $\frac{63}{4} \times \frac{4}{9} = 7$ kurtas. **[1 Mark]**

62. Distribute left side: $\frac{5}{6}x + \frac{15}{30} = \frac{5}{6}x + \frac{1}{2}$ **[1 Mark]**

Simplify fraction: $\frac{15}{30} = \frac{1}{2}$. Equation is $\frac{5}{6}x + \frac{1}{2} = \frac{5}{6}x + \frac{1}{2}$. **[1 Mark]**

Conclusion: This is an algebraic identity; x can be *any* rational number. **[1 Mark]**

63. $x = 2.35757\ldots \Longrightarrow 10x = 23.5757\ldots$ **[1 Mark]**

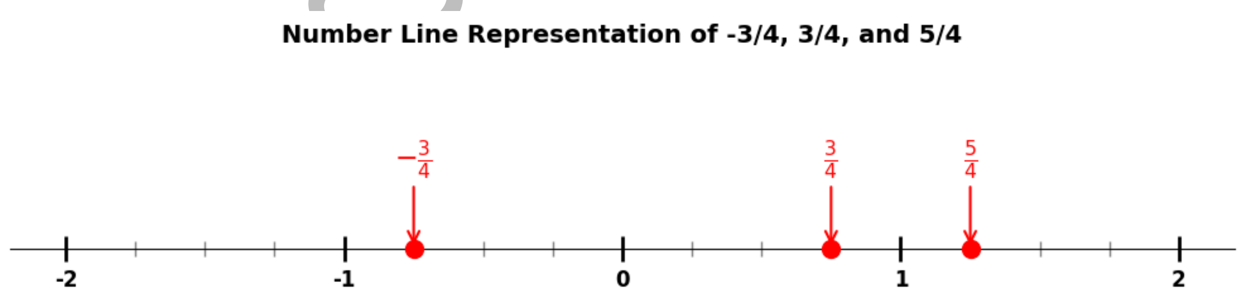
Multiply original by 1000: $1000x = 2357.5757\ldots$ **[1 Mark]**

Subtract: $990x = 2334 \Longrightarrow x = \frac{2334}{990} = \frac{389}{165}$. **[1 Mark]**

64. Drawing a horizontal line with integers marked. **[1 Mark]**

Dividing distance between integer units into 4 equal segments. **[1 Mark]**

Correctly plotting $\frac{3}{4}$, $\frac{5}{4}$, and $-\frac{3}{4}$. **[1 Mark]**



65. Denominator 125 = 5^3. It has only prime factor 5, so it terminates. **[1 Mark]**

Multiply numerator and denominator by 2^3 (8): $\frac{18\times8}{125\times8} = \frac{144}{1000}$. **[1 Mark]**

Evaluates to 0.144, which is exactly 3 decimal places. **[1 Mark]**

66. Algebra: Let $x = 0.999\ldots$, $10x = 9.999\ldots$, $10x - x = 9 \Longrightarrow 9x = 9 \Longrightarrow x = 1$. **[1.5 Marks]**

Concept: Terminating decimals can have an alternative representation with infinite trailing 9s, proving non-uniqueness. **[1.5 Marks]**

67. Bakhśhālī MS represented zero physically as an empty placeholder (bold dot). **[1.5 Marks]**

    Brahmagupta gave zero actual arithmetic *rules* (like $a - a = 0$), transforming it into an operational number. **[1.5 Marks]**

68. Common denominator 12. $b = \frac{10}{12}$. **[1 Mark]**

    $a + b = \frac{7}{12} + \frac{10}{12} = \frac{17}{12}$. **[1 Mark]**

    $b - a = \frac{10}{12} - \frac{7}{12} = \frac{3}{12} = \frac{1}{4}$. **[1 Mark]**

69. Identity used: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$. **[1 Mark]**

    Substituting values: $0^2 = x^2 + y^2 + z^2 + 2(0) \Longrightarrow x^2 + y^2 + z^2 = 0$. **[1 Mark]**

    Since squares of rationals are non-negative, sum is 0 iff $x = 0$, $y = 0$, $z = 0$ simultaneously. **[1 Mark]**

70. *Śhūnyatā* (emptiness) was a profound state of meditation/yoga (emptying mind of *vṛttis*). **[1.5 Marks]**

    Because Indian thinkers philosophically revered 'emptiness', they possessed the framework to embrace 'nothingness', allowing its mathematical acceptance as zero. **[1.5 Marks]**

SECTION D (5-Mark Questions)

71. **Assumption:** Assume $\sqrt{2} = \frac{p}{q}$ in simplest form (co-prime). **[1 Mark]**

    Squaring: $2 = \frac{p^2}{q^2} \Longrightarrow 2q^2 = p^2$. **[1 Mark]**

    Deduction 1: p^2 is even, so p must be even. Let $p = 2k$. **[1 Mark]**

    Substitution: $2q^2 = (2k)^2 = 4k^2 \Longrightarrow q^2 = 2k^2$. **[1 Mark]**

    Deduction 2 & Contradiction: q^2 is even, so q is even. Both share a factor of 2, contradicting they are co-prime. Hence $\sqrt{2}$ is irrational. **[1 Mark]**

72. **Assumption:** Assume $\sqrt{3} = \frac{p}{q}$ in simplest form (co-prime). **[1 Mark]**

    Squaring: $3 = \frac{p^2}{q^2} \Longrightarrow p^2 = 3q^2$. **[1 Mark]**

    Deduction 1: p^2 is a multiple of 3, so p must be a multiple of 3. Let $p = 3k$. **[1 Mark]**

    Substitution: $3q^2 = (3k)^2 = 9k^2 \Longrightarrow q^2 = 3k^2$. **[1 Mark]**

Deduction 2 & Contradiction: q^2 is a multiple of 3, so q is a multiple of 3. Both share factor 3. Contradiction. $\sqrt{3}$ is irrational. **[1 Mark]**

73. **Step 1:** Mark origin $O = 0$ and point A at exactly 1 unit distance to the right on the number line. **[1 Mark]**

    Step 2: Construct perpendicular line segment AB at A of exactly 1 unit length. **[1 Mark]**

    Step 3: Join O to B to form right-angled $\triangle OAB$. **[1 Mark]**

    Step 4: By Pythagoras: $OB^2 = OA^2 + AB^2 = 1^2 + 1^2 = 2 \Longrightarrow OB = \sqrt{2}$. **[1 Mark]**

    Step 5: Place compass at O, pencil at B. Draw arc down to intersect number line at P. $OP = \sqrt{2}$. **[1 Mark]**

74. **Proof Part 1:** Add a to $a < b$: $2a < a + b \Longrightarrow a < \frac{a+b}{2}$. **[1.5 Marks]**

    Proof Part 2: Add b to $a < b$: $a + b < 2b \Longrightarrow \frac{a+b}{2} < b$. Combines to $a < \frac{a+b}{2} < b$. **[1.5 Marks]**

    Verification Setup: For $\frac{2}{3}$ and $\frac{3}{5}$, use denominator 15 $\Longrightarrow \frac{10}{15}$ and $\frac{9}{15}$. **[1 Mark]**

    Verification Result: Average is $\frac{\frac{10}{15}+\frac{9}{15}}{2} = \frac{19}{30}$. $\frac{19}{30}$ lies between $\frac{18}{30}$ and $\frac{20}{30}$. **[1 Mark]**

75. **Eq 1:** Let $x = 2.453737\ldots$ **[0.5 Marks]**

    Eq 2: Multiply by 100 to shift non-repeating part: $100x = 245.3737\ldots$ **[1.5 Marks]**

    Eq 3: Multiply x by 10000 to shift full repeating block: $10000x = 24537.3737\ldots$ **[1.5 Marks]**

    Subtraction: $10000x - 100x = 24537 - 245 \Longrightarrow 9900x = 24292$. **[1 Mark]**

    Final Fraction: $x = \frac{24292}{9900} = \frac{6073}{2475}$. **[0.5 Marks]**

76. **Initial setup:** $a = \frac{7}{12}$ and $b = \frac{10}{12}$. **[1 Mark]**

    Logic: Between 7 and 10, there are 2 integers. For $n = 5$ numbers, we need gap

    $k_2 - k_1 > 6$. **[1.5 Marks]**

    Scaling: Multiply numerator and denominator by 6. **[1 Mark]**

    New fractions: $a = \frac{42}{72}$, $b = \frac{60}{72}$. **[0.5 Marks]**

    Selection: Any five distinct rational numbers like $\frac{43}{72}, \frac{44}{72}, \frac{45}{72}, \frac{46}{72}, \frac{47}{72}$. **[1 Mark]**

77. **Long Division:** $1 \div 7$ yields $0.\overline{142857}$. **[2 Marks]**

Evaluating $\frac{3}{7}$: $3 \times 142857 = 428571 \Rightarrow 0.\overline{428571}$. **[1.5 Marks]**

Evaluating $\frac{5}{7}$: $5 \times 142857 = 714285 \Rightarrow 0.\overline{714285}$. **[1.5 Marks]**

78. $\mathbb{N}$ (Natural): Counting numbers $\{1,2...\}$, limited by subtraction. **[1 Mark]**

$\mathbb{Z}$ (Integers): Added zero and negative numbers (Debts) to resolve subtraction. **[1 Mark]**

$\mathbb{Q}$ (Rational): Added fractions $\frac{p}{q}$ for parts of wholes, creating a dense line. **[1 Mark]**

$\mathbb{I}$ (Irrational): Measurable lengths (diagonals) that defied $\frac{p}{q}$ ratios. **[1 Mark]**

$\mathbb{R}$ (Real): Complete union of $\mathbb{Q}$ and $\mathbb{I}$ forming a continuous physical line. **[1 Mark]**

79. **Setup:** Let decimal be $0.d_1d_2d_3d_4$ ($d_4 \neq 0$). Fraction is $\frac{p}{10^4}$. **[1 Mark]**

Divisibility of p: Since $d_4 \neq 0$, p doesn't end in 0, hence p is not divisible by 10. **[1 Mark]**

Denominator analysis: $10^4 = 2^4 \times 5^4$. **[1 Mark]**

Conclusion: Since p is not divisible by 10, it cannot have BOTH factors 2 and 5. Upon reduction, at least ONE full factor (2^4 or 5^4) MUST remain to ensure it ends strictly at 4 decimal places. **[2 Marks]**

80. **Explanation:** π is irrational, so its decimal never ends/repeats. A finite fraction must terminate or repeat, thus no single fraction perfectly captures it. **[2 Marks]**

Setup: Sum of first 4 terms $= 4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\right)$. **[1 Mark]**

Common denominator: 105. Expression is $4 \times \left(\frac{105-35+21-15}{105}\right)$. **[1 Mark]**

Final Result: $4 \times \left(\frac{76}{105}\right) = \frac{304}{105} \approx 2.895$. **[1 Mark]**

SECTION E (Case Study Based Questions)

81. **Case Study 1: The Ishango Bone**

(i) They are Prime Numbers (only divisible by 1 and themselves). **[1 Mark]**

(ii) One-to-one correspondence. **[1 Mark]**

(iii) **Choice 1:** The next two prime numbers are 23 and 29. **[1 Mark for 23, 1 Mark for 29 = 2 Marks]**
OR

Choice 2: $17 \times 2 = 34$. **[1 Mark for expression, 1 Mark for 34 notches = 2 Marks]**

82. **Case Study 2: Brahmagupta's Ledger**

(i) A fortune of 7 (since $a - 0 = a$). **[1 Mark]**

(ii) The set of Integers. Symbol: $\mathbb{Z}$. **[0.5 Marks for name, 0.5 Marks for symbol = 1 Mark]**

(iii) **Choice 1:** $(-15) + (-10) = -25$. **[1 Mark for setup, 1 Mark for -25 = 2 Marks]**
OR

Choice 2: $5 \times (-4) = -20$. The final state is a Debt. **[1 Mark for expression, 1 Mark for Debt state = 2 Marks]**

83. **Case Study 3: The Search for Pi (π)**

(i) Because π is an irrational number. **[1 Mark]**

(ii) Irrational Numbers. **[1 Mark]**

(iii) **Choice 1:** It will terminate. Prime factors of 1250 are 2×5^4, consisting exclusively of 2s and 5s. **[1 Mark for 'terminate', 1 Mark for justification = 2 Marks]**
OR

Choice 2: The first three terms are $1, -\frac{1}{3}$, and $+\frac{1}{5}$. **[2 Marks for all three correct, 1 mark for partially correct]**

84. **Case Study 4: Cyclic Decimals**

(i) The possible remainders are 1, 2, 3, 4, 5, and 6. **[1 Mark]**

(ii) Because there is a limited number of possible remainders (6), a remainder must eventually repeat. **[1 Mark]**

(iii) **Choice 1:** $\frac{2}{7} = 0.\overline{285714}$. **[2 Marks for correctly shifted sequence]**
OR

Choice 2: Since there are 6 digits in the repeating block, you must multiply x by 10^6 to shift exactly one full cycle to the left. **[1 Mark for 10^6, 1 Mark for explanation = 2 Marks]**

85. **Case Study 5: The Revolution of Śhūnya**

(i) A profound state of perfect stillness and tranquility. **[1 Mark]**

(ii) The *Bakhśhālī Manuscript*. **[1 Mark]**

(iii) **Choice 1:** Babylonians used it merely as an empty placeholder column, but Brahmagupta established arithmetic rules so it was treated as an operational number. **[1 Mark for placeholder, 1 Mark for operational number = 2 Marks]**
OR

Choice 2: Mathematically, zero is the result of subtracting a number from itself: $a - a = 0$.
[2 Marks for correct algebraic equation]