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THE WORLD OF NUMBERS
CLASS 9
EXTRA QUESTIONS/QUESTION BANK

Advanced Competency-Based Question Bank: The World of Numbers
CBSE Grade 9 Mathematics (NEP 2020 Guidelines)
Focus: Real-world Applications, Higher Order Thinking Skills

SECTION A: 1-Mark Questions (40 Questions)

Multiple Choice Questions (Q1 - Q20)

(Hints provided for all questions. Tagged by difficulty.)

1. A drone operates at an altitude of 150 meters above ground. It descends 45 meters to take a photo. Which integer operation models this final altitude? [**Easy**] [**Competency-Based**]
   A) $(+150) + (+45)$
   B) $(+150) - (+45)$
   C) $(-150) + (-45)$
   D) $(-150) - (-45)$
   Hint: "Above ground" is positive; "descending" is a reduction or subtraction of a positive height.

2. A digital caliper measures a machine part to exactly 2.0625 cm. Based on this terminating decimal, what must be true about the fraction $\frac{p}{q}$ (in lowest terms) representing this length? [**Difficult**] [**Competency-Based**]
   A) The denominator has prime factors of 3 and 5.
   B) The denominator has only prime factors of 2 and/or 5.
   C) The numerator is a prime number.
   D) The denominator is a multiple of 7.
   Hint: Terminating decimals require denominators that are powers of 10, meaning prime factors of 2 and 5.

3. An urban planner designs a square fountain with an exact area of 12 square meters. The side length of this fountain is a number that is: [**Easy**] [**Competency-Based**]
   A) Rational and terminating
   B) Rational and repeating
   C) An Integer
   D) Irrational
   Hint: The side length is $\sqrt{12}$. Is 12 a perfect square?

4. A software algorithm distributes server load using the fraction $\frac{1}{7}$, creating the repeating decimal $0.\overline{142857}$. What will be the 40th digit after the decimal point? [**HOTS**] [**Competency-Based**]
   A) 1
   B) 4
   C) 8

D) 7
Hint: The repeating block has 6 digits. Divide 40 by 6 to find the remainder, which indicates the position.

5. A trader experiences a loss of 15 per share on 20 shares of stock. What is the mathematical product representing the total financial impact? [**Difficult**] [**Competency-Based**]
A) $(+15) \times (+20) = +300$
B) $(-15) \times (+20) = -300$
C) $(-15) \times (-20) = +300$
D) $(+15) \times (-20) = -300$
Hint: A loss is a negative integer (debt), and the number of shares is a positive integer (fortune).

6. Which of the following practical divisions will produce a repeating decimal, potentially causing rounding errors in a basic calculator? [**Difficult**] [**Competency-Based**]
A) Cutting a 1-meter wire into 16 equal pieces $\left(\frac{1}{16}\right)$
B) Dividing a 1-liter solution into 25 equal vials $\left(\frac{1}{25}\right)$
C) Sharing 100 equally among 6 people $\left(\frac{100}{6}\right)$
D) Splitting a 5 kg bag of flour into 4 equal portions $\left(\frac{5}{4}\right)$
Hint: Simplify $\frac{100}{6}$ to $\frac{50}{3}$. Check the prime factors of all denominators.

7. Two laser sensors are positioned at 3.2 mm and 3.3 mm on a track. To calibrate the system, a target must be placed exactly halfway between them. What is the target's position? [**Easy**] [**Competency-Based**]
A) 3.21 mm
B) 3.25 mm
C) 3.35 mm
D) 3.15 mm
Hint: Use the average formula $(a + b)/2$ to find the midpoint.

8. A machine learning model calculates a probability weight as exactly $0.99999...$ (repeating infinitely). How does the math processor simplify this value? [**HOTS**] [**Competency-Based**]
A) It rounds it down to 0.99.
B) It stores it as exactly 1.
C) It stores it as an irrational number.
D) It creates an infinite memory loop.
Hint: $0.\overline{9}$ is mathematically identical to the integer 1.

9. To build a stable support bracket, a carpenter needs a diagonal length of exactly $\sqrt{13}$ cm. Which two perpendicular integer lengths should he use for the base and height? [**Difficult**] [**Competency-Based**]
A) 1 cm and 12 cm
B) 2 cm and 3 cm
C) 3 cm and 4 cm
D) 4 cm and 5 cm
Hint: Check the Pythagorean theorem $a^2 + b^2 = (\sqrt{13})^2 = 13$.

10. A financial audit finds a discrepancy of -500 dollars. If the auditor wants to report the total magnitude of the error without regard to whether it was a surplus or deficit, they must use: [**Easy**] [**Competency-Based**]

A) The Additive Inverse
B) Absolute Value
C) The Distributive Property
D) Density of Rationals
Hint: This concept measures physical distance or magnitude from zero.

11. A random number generator outputs $0.5050050005\ldots$ (increasing the number of zeros between each 5). What category does this decimal belong to? [**HOTS**] [**Competency-Based**]
A) Rational, because it follows a logical pattern.
B) Irrational, because it is non-terminating and non-repeating.
C) Integer, because it only uses 0 and 5.
D) Rational, because it can be written as a fraction.
Hint: A changing pattern means there is no fixed, looping sequence.

12. In an electrical circuit, a $+12$V battery is connected in series with a -12V power drop. What is the net voltage, illustrating the concept of additive inverses? [**Easy**] [**Competency-Based**]
A) $+24$V
B) -24V
C) 0V
D) $+144$V
Hint: Adding a fortune and an equal debt results in nothingness.

13. A chemist scales down a formula, requiring $\frac{21}{14}$ grams of a compound. What type of decimal will this form on the digital scale? [**Difficult**] [**Competency-Based**]
A) Terminating
B) Pure repeating
C) General repeating
D) Irrational
Hint: Simplify the fraction first (divide by 7) before checking the denominator.

14. An astrological chart rotates using a base fraction of $\frac{6}{7}$. Knowing $\frac{1}{7} = 0.\overline{142857}$, what is the first digit after the decimal point in the expansion of $\frac{6}{7}$? [**HOTS**] [**Competency-Based**]
A) 1
B) 4
C) 7
D) 8
Hint: Since $6 \times 0.14 = 0.84$*, the cyclic sequence must start with the largest digit.*

15. A surveyor maps a property boundary from coordinate $(0,0)$ to $(5,12)$ on a grid. The direct distance is: [**Easy**] [**Competency-Based**]
A) 17 units (Rational)
B) $\sqrt{17}$ units (Irrational)
C) 13 units (Rational)
D) $\sqrt{119}$ units (Irrational)
Hint: Use the Pythagorean theorem: $\sqrt{5^2 + 12^2}$.

16. A cryptography algorithm uses the fraction $\frac{1}{19}$. What is the maximum possible number of digits in its repeating decimal block? [**Difficult**] [**Competency-Based**]
A) 9

B) 18
C) 19
D) 20
Hint: The maximum number of non-zero remainders when dividing by prime p *is* $p-1$.

17. A mechanic needs a wrench sized exactly 0.625 inches. What is this measurement as a simplified rational fraction? [**Easy**] [**Competency-Based**]
A) $\frac{5}{8}$
B) $\frac{3}{4}$
C) $\frac{2}{3}$
D) $\frac{7}{12}$
Hint: Write $\frac{625}{1000}$ *and divide numerator and denominator by 125.*

18. An architect looks at a list of beam lengths: $\sqrt{4}$m, $\sqrt{6}$m, $\sqrt{9}$m, $\sqrt{12}$m. How many of these lengths are rational and can be measured exactly without approximation? [**Difficult**] [**Competency-Based**]
A) 1
B) 2
C) 3
D) 4
Hint: Only perfect squares have rational square roots.

19. Using Madhava's infinite series $4 \times (1 - \frac{1}{3} + \frac{1}{5} - \cdots)$ to approximate π, what is the calculated value if a computer uses only the first two terms in the bracket? [**HOTS**] [**Competency-Based**]
A) $2.66\ldots$
B) $3.14\ldots$
C) 4.00
D) $1.33\ldots$
Hint: Evaluate $4 \times \left(1 - \frac{1}{3}\right) = 4 \times \left(\frac{2}{3}\right)$.

20. A bank reverses an erroneous penalty charge of 35 on a customer's account. Mathematically, subtracting this debt is represented as $-(-\text{Rs. } 35)$. What is the effect? [**Difficult**] [**Competency-Based**]
A) The account balance drops by 35.
B) The account balance increases by 35.
C) The balance remains unchanged.
D) The balance goes to zero.
Hint: Subtracting a negative acts mathematically like adding a positive.

Objective Type Questions (Q21 - Q30)

Fill in the blanks (Q21 - Q25)

21. In a physics simulation, the exact diagonal distance across a 2×2 meter grid is measured as $\sqrt{8}$. This distance cannot be written as $\frac{p}{q}$, meaning it belongs to the set of numbers ….. [**Easy**] [**Competency-Based**]
Hint: Numbers that have non-terminating, non-repeating decimals.

22. An accountant logs a 15% loss as -0.15 and a 20% gain as $+0.20$. Using the ancient Indian terminology from Brahmagupta, the loss is a *Ṛiṇa* and the gain is a …... [**Easy**] [**Competency-Based**]
Hint: What is the Sanskrit term for a fortune or asset?

23. A smart meter records water flow at $0.777...$ liters per second. To use this in an algebraic formula, an engineer converts this pure repeating decimal into the exact rational fraction ……. [**Difficult**] [**Competency-Based**]
Hint: Let $x = 0.\overline{7}$, multiply by 10, and subtract.

24. The mathematical principle that guarantees an engineer can always calculate a precise fractional tolerance exactly halfway between 0.05 cm and 0.06 cm is known as the ……. property of rational numbers. [**Difficult**] [**Competency-Based**]
Hint: It means the numbers are packed infinitely close together with no empty spaces between them.

25. When a cyclic redundancy check uses the decimal expansion of $\frac{3}{7}$, the resulting sequence of digits is simply a …….. shift of the digits found in $\frac{1}{7}$. [**HOTS**] [**Competency-Based**]
Hint: The digits 142857 loop around in a circle.

True or False (Q26 - Q30)

26. A physics engine calculating orbital mechanics using $\pi = \frac{22}{7}$ is utilizing an exact irrational value for perfect accuracy. [**Easy**] [**Competency-Based**]
Hint: Is $\frac{22}{7}$ in the form of $\frac{p}{q}$? Does it terminate or repeat?

27. If a deep-sea probe descends to -850m and an airplane flies at $+850$m, their absolute distances from sea level are perfectly equal. [**Easy**] [**Competency-Based**]
Hint: Absolute value $|x|$ ignores the direction (sign) and only looks at magnitude.

28. When a barista splits exactly 10 liters of milk equally into 3 containers, the decimal representation of the volume in each container will terminate. [**Difficult**] [**Competency-Based**]
Hint: Evaluate $\frac{10}{3}$. Does the denominator have only prime factors of 2 and 5?

29. The geometric proof showing that a square plaza cannot have both a rational side length and a rational diagonal relies on the logical method called Proof by Contradiction. [**HOTS**] [**Competency-Based**]
Hint: How did Hippasus prove that $\sqrt{2}$ was not a fraction?

30. According to integer arithmetic applied to a ledger, removing 4 separate debts of 20 each results in a net mathematical gain of 80. [**Difficult**] [**Competency-Based**]
Hint: Evaluate $(-4) \times (-20)$.

Assertion-Reasoning Type Questions (Q31 - Q40)

Options for Q31-Q40:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

31. **Assertion (A):** A seismologist records an earthquake wave frequency as $0.2020020002...$ (increasing the number of zeros) and correctly categorizes it as an irrational number.
Reason (R): The number has a decimal expansion that is non-terminating and non-repeating. [**HOTS**] [**Competency-Based**]
Hint: Does a growing pattern count as a repeating cyclic block?

32. **Assertion (A):** A store owner calculates net daily profit using the concept: "A fortune minus zero is a fortune."
Reason (R): Subtracting zero from any rational number a yields 0. [**Easy**] [**Competency-Based**]
Hint: Does $a - 0 = 0$ or does $a - 0 = a$?

33. **Assertion (A):** A factory produces a metal rod of length exactly $\frac{27}{50}$ meters. A digital quality checker will read this as a terminating decimal.
Reason (R): The prime factorization of 50 (2×5^2) contains only prime factors of 2 and 5. [**Difficult**] [**Competency-Based**]
Hint: How do factors of 2 and 5 affect decimal termination?

34. **Assertion (A):** The GPS coordinates relying on π and $\frac{22}{7}$ point to the exact same continuous location on the Real Number line.
Reason (R): π is an irrational number, while $\frac{22}{7}$ is a rational number. [**Difficult**] [**Competency-Based**]
Hint: Is $\frac{22}{7}$ an exact value for π or just a close approximation?

35. **Assertion (A):** A company tracks a stock price drop of 18 followed by a drop of 14, recording a total change of -32.
Reason (R): The addition of two debts results in a larger debt. [**Easy**] [**Competency-Based**]
Hint: Evaluate $(-18) + (-14)$.

36. **Assertion (A):** A machinist can cut a precise circular plate with a radius of exactly $\sqrt{9}$ cm using a standard rational measuring tape.
Reason (R): $\sqrt{9}$ simplifies to the integer 3, which is a rational number. [**Easy**] [**Competency-Based**]
Hint: Are all square roots irrational?

37. **Assertion (A):** It is impossible to program a robot to move to a location exactly halfway between 2.5 meters and 2.6 meters.
Reason (R): Rational numbers are dense, meaning there is an infinite number of rational points between any two distinct rational numbers. [**Difficult**] [**Competency-Based**]
Hint: Does the density property prevent finding a midpoint, or does it guarantee one?

38. **Assertion (A):** A server calculating bandwidth allocation using $\frac{1}{19}$ will eventually produce a repeating loop of digits.
Reason (R): When dividing by 19, there are only 18 possible non-zero remainders, forcing the sequence to eventually repeat. [**HOTS**] [**Competency-Based**]
Hint: Think about why fractions become repeating decimals (the pigeonhole principle).

39. **Assertion (A):** A freezer operating at $|-30°C|$ represents a greater deviation from freezing (0°C) than a fridge operating at $|-5°C|$.
Reason (R): Absolute value represents distance from zero, so $|-30| = 30$, indicating a larger

magnitude. [**Difficult**] [**Competency-Based**]
Hint: Does absolute value measure the size of the deviation?

40. **Assertion (A):** In high-frequency trading algorithms, $0.999 \ldots$ and 1.0 are processed as exactly the same mathematical value.
Reason (R): Any terminating decimal has an alternative equivalent form using infinite repeating 9s. [**HOTS**] [**Competency-Based**]
Hint: Does $0.\overline{9} = 1$ *mathematically?*

SECTION B: 2-Mark Questions (20 Questions)

41. A mining elevator is at -25 meters below the surface. It descends another 18 meters to reach a lower shaft, then ascends 12 meters for maintenance. Model this using integers and find the final depth. [**Easy**] [**Competency-Based**]
Hint: "Descend" means subtract, "ascend" means add.

42. An optical lens requires a thickness exactly halfway between 0.4 cm and 0.45 cm. Calculate this exact rational thickness as a fraction in simplest form. [**Difficult**] [**Competency-Based**]
Hint: Convert decimals to fractions first $\left(\frac{2}{5} \text{ and } \frac{9}{20}\right)$ *, then find their average.*

43. A farmer measures the diagonal of a rectangular plot, calculating it as $\sqrt{12}$ kilometers. Is this distance rational or irrational? Simplify the radical to justify your answer. [**Easy**] [**Competency-Based**]
Hint: Can $\sqrt{12}$ *be broken down into* $\sqrt{4 \times 3}$*?*

44. A laboratory scale measures a gold sample at exactly 0.145 grams. Convert this digital reading into a precise rational fraction $\frac{p}{q}$ in simplest form. [**Difficult**] [**Competency-Based**]
Hint: Write as $\frac{145}{1000}$ *and simplify by dividing by a common factor.*

45. Explain practically why absolute value is necessary when calculating the total length of wire needed to run from a ceiling junction at $+4$m down to a floor outlet at -1m. [**HOTS**] [**Competency-Based**]
Hint: Wire length cannot be negative. Use $|+4-(-1)|$.

46. A 3D printer rotates its base plate by $\frac{7}{40}$ of a circle per layer. Will the decimal equivalent of this rotation terminate or repeat? Justify without long division. [**Difficult**] [**Competency-Based**]
Hint: Look at the prime factors of 40.

47. A bookstore cancels 4 delayed book orders, each representing a revenue debt of 300. Using integer multiplication, show the net effect on the bookstore's ledger. [**Easy**] [**Competency-Based**]
Hint: Cancelling (taking away) is negative. A debt is negative. $(-4) \times (-300)$.

48. A digital tuner uses a frequency ratio of $0.\overline{45}$. Convert this repeating decimal into a rational fraction to determine the lowest integer ratio of waves. [**HOTS**] [**Competency-Based**]
Hint: Let $x = 0.4545\ldots$*, multiply by 100, subtract* x*, and simplify.*

49. On a corporate ledger, Division A is at $+5.2$ million and Division B is at -1.8 million. What is the absolute financial distance (spread) between the two divisions? [**Easy**] [**Competency-Based**]
Hint: Distance between a *and* b *is* $|a-b|$.

50. An IT firm generates passwords using the cyclic sequence from $\frac{1}{7}$. If the base sequence is 142857, write the exact cyclic shifts produced when evaluating $\frac{3}{7}$ and $\frac{4}{7}$. [**Difficult**] [**Competency-Based**]
Hint: $\frac{3}{7}$ *starts with the 3rd lowest digit.* $\frac{4}{7}$ *starts with the 4th lowest digit.*

51. Prove whether the product of a rational length (1.5 m) and an irrational width ($\sqrt{2}$ m) results in a rational or irrational area. [**HOTS**] [**Competency-Based**]
Hint: Use proof by contradiction. Assume $1.5 \times \sqrt{2} = \frac{p}{q}$.

52. A baker mixes $3\frac{1}{3}$ cups of whole wheat flour and $2\frac{1}{4}$ cups of white flour. Find the total volume as an improper rational fraction. [**Easy**] [**Competency-Based**]
Hint: Convert to improper fractions $\left(\frac{10}{3} \text{ and } \frac{9}{4}\right)$ *and find a common denominator.*

53. A data compression algorithm requires evaluating the expression: $\frac{5}{12} \times \left(-\frac{4}{15}\right)$. Find the exact rational output. [**Difficult**] [**Competency-Based**]
Hint: Multiply numerators and denominators, then simplify.

54. A textile machine leaves a repeating flaw every $0.6111...$ meters. Express this general repeating decimal ($0.6\overline{1}$) as a fraction of a meter in simplest form. [**HOTS**] [**Competency-Based**]
Hint: Let $x = 0.611...$ *Multiply by 10 to shift the 6, then by 10 again to shift the 1. Subtract.*

55. Evaluate the arithmetic rule $a - (-b) = a + b$ using a real-world scenario where a company has a balance of 500 and the bank reverses an overdraft fee of 45. [**Easy**] [**Competency-Based**]
Hint: Reversing a fee is subtracting a debt: $500 - (-45)$.

56. A sports car accelerates, covering a distance mapped by the equation $d = \frac{13}{20}$ km. Convert this directly into a terminating decimal without using a calculator. [**Difficult**] [**Competency-Based**]
Hint: Multiply numerator and denominator by 5 to make the denominator 100.

57. The Bakhśhālī manuscript used a dot (bindu) as a placeholder. Explain practically how modern computer programming utilizes Brahmagupta's operational rules for zero (e.g., $a \times 0 = 0$) differently than a mere placeholder. [**HOTS**] [**Competency-Based**]
Hint: A placeholder just visually separates digits (like 101). Brahmagupta's rules allow a program to calculate that any variable multiplied by zero wipes out the value.

58. Evaluate the absolute expression: $|-8| + |-12| - |+5|$. [**Easy**] [**Competency-Based**]
Hint: Turn all values positive before adding and subtracting.

59. Determine if the rational numbers $\frac{-3}{8}$ and $\frac{105}{-280}$ are equivalent, showing the cross-multiplication or simplification steps. [**Difficult**] [**Competency-Based**]
Hint: Simplify $\frac{105}{280}$ *by dividing by 5, then by 7. Or use* $ad = bc$.

60. A recipe calls for $\frac{5}{2}$ cups of sugar, but a chef only wants to make $\frac{1}{3}$ of the batch. Calculate the exact fraction of sugar needed. [**Easy**] [**Competency-Based**]
Hint: Multiply the two fractions.

SECTION C: 3-Mark Questions (20 Questions)

61. A warehouse tracks inventory changes for a specific electronic part. On Monday, they receive 350 units. On Tuesday, they ship out 180 units. On Wednesday, they ship out another 220 units. (i) Write an integer equation modeling this sequence. (ii) Calculate the final net change in inventory. (iii) Is the final state a "fortune" (surplus) or "debt" (deficit)? [**Easy**] [**Competency-Based**]
Hint: Receiving is positive, shipping is negative.

62. A financial modeling software uses a growth multiplier of $2.3\overline{15}$. To avoid floating-point errors, the software must convert this into a precise fractional ratio $\frac{p}{q}$. Determine this fraction. [**HOTS**] [**Competency-Based**]
Hint: Let $x = 2.31515...$ *Multiply by 10 to isolate the repeating block, then by 1000 to shift the full block.*

63. A railway engineer maps three stations on a straight line: Station Alpha at -4.5 km, Station Beta at 1.5 km, and Station Gamma at 6.5 km. (i) Find the exact halfway coordinate between Alpha and Beta. (ii) Find the exact halfway coordinate between Beta and Gamma. [**Difficult**] [**Competency-Based**]
Hint: Use the average formula $\frac{a+b}{2}$ *for both pairs.*

64. A civil engineer needs to verify if a diagonal truss of $\sqrt{7}$ meters can ever be cut precisely using a standard rational measuring tape. Prove, using the method of contradiction, that $\sqrt{7}$ is an irrational number. [**HOTS**] [**Competency-Based**]
Hint: Assume $\sqrt{7} = \frac{p}{q}$. *Square both sides. Show* p^2 *is a multiple of 7, meaning* p *is a multiple of 7, which leads to* q *also being a multiple of 7.*

65. A bakery portions its giant cake into 15 slices $\left(\frac{1}{15}\right)$ and its standard cake into 10 slices $\left(\frac{1}{10}\right)$. (i) Without long division, determine which portion size results in a terminating decimal and which results in a repeating decimal. (ii) Justify your answer using prime factorization. [**Difficult**] [**Competency-Based**]
Hint: Analyze the prime factors of denominators 15 and 10.

66. A submarine ascends $20\frac{1}{4}$ meters toward the surface. It then dives $8\frac{1}{2}$ meters to avoid a ship, and ascends again by $5\frac{3}{8}$ meters. Calculate its final net depth change as a rational fraction. [**Easy**] [**Competency-Based**]
Hint: Convert all to improper fractions with a common denominator of 8.

67. In an engineering simulation, fluid flows are represented as fractions of a total. Three intake pipes provide flows of $x = 0.\overline{6}$, $y = 0.25$, and $z = 0.08\overline{3}$. (i) Convert x and z into fractions. (ii) Show whether their sum exactly equals a full flow of 1. [**HOTS**] [**Competency-Based**]
Hint: Convert $0.08333...$ *by shifting the decimal* $(100x$ *and* $1000x)$.

68. A weather station converts temperatures using the distributive property formula: $C = \frac{5}{9}(F - 32)$. If the Fahrenheit temperature F is 77°, calculate the Celsius temperature C. Show the step-by-step rational multiplication. [**Easy**] [**Competency-Based**]
Hint: Subtract 32 from 77 first, then multiply by $\frac{5}{9}$.

69. A graphics rendering engine calculates a pixel coordinate as $\frac{1017}{3051}$ cm. The system optimizer wants to simplify this fraction. Prove algebraically or through simplification that this fraction is exactly equivalent to $\frac{1}{3}$. [**Difficult**] [**Competency-Based**]
Hint: Multiply 1017 by 3 to verify equivalence.

70. An accounting firm evaluates a client's portfolio: Asset A = -400 (Debt), Asset B = $+900$ (Fortune), Asset C = -1500 (Debt). Calculate the value of $A \times (B + C)$ using Brahmagupta's rules of integers. State whether the final result is a Fortune or Debt. [**Easy**] [**Competency-Based**]
Hint: Add B and C first. Then multiply a debt by a debt.

71. The repeating block of $\frac{1}{7}$ is 142857. A cryptographer is creating an encryption key that shifts this sequence. Evaluate $\frac{2}{7}$ and $\frac{5}{7}$ relying only on the cyclic shift property (no long division). [**Difficult**] [**Competency-Based**]
Hint: $2/7$ *starts with the 2nd lowest digit (2).* $\frac{5}{7}$ *starts with the 5th lowest digit (7).*

72. To calculate load distribution, an architect evaluates the expression $\left(-\frac{4}{7}\right) \times \left(\frac{14}{3} + \frac{21}{8}\right)$. Solve this using the distributive property for rational numbers. [**HOTS**] [**Competency-Based**]
Hint: Distribute the multiplication first: multiply $\left(-\frac{4}{7}\right)$ *with both terms inside the bracket, then add.*

73. A chemical vat drains at a rate of $\frac{2}{5}$ liters/sec. A supply line fills it at $\frac{3}{4}$ liters/sec. If both are open, find the net change in liquid volume per second. Represent this conceptually as a rational fraction. [**Difficult**] [**Competency-Based**]
Hint: Fill is positive $\left(+\frac{3}{4}\right)$*, drain is negative* $\left(-\frac{2}{5}\right)$*. Find a common denominator (20).*

74. Explain practically how the non-uniqueness of decimal representations ($1.000\ldots = 0.999\ldots$) applies to a highly sensitive atomic clock measuring exactly 1 second, using algebraic proof to support your reasoning. [**HOTS**] [**Competency-Based**]
Hint: Show the $10x - x$ *proof for* $0.\overline{9}$*. Physically,* $0.999\ldots$ *seconds is indistinguishable from 1 second because they are mathematically equal.*

75. A researcher notes that Aryabhata approximated π as $\frac{3927}{1250}$. Calculate the decimal value of this approximation. Explain why this specific decimal proves the fraction is rational, even though π itself is irrational. [**Difficult**] [**Competency-Based**]
Hint: Divide 3927 by 1250. Because it is a terminating decimal, it is by definition a rational number.

76. Evaluate using integer laws: $(-25) \times (-4) \div (-10)$. [**Easy**] [**Competency-Based**]
Hint: Do multiplication first (negative × negative), then division.

77. Find exactly three rational numbers between $\frac{-2}{3}$ and $\frac{1}{5}$. [**Difficult**] [**Competency-Based**]
Hint: Use common denominator 15.

78. A machine cuts a board into $2.1\overline{4}$ meters. Convert this general repeating decimal into a rational fraction. [**HOTS**] [**Competency-Based**]
Hint: Shift decimal past the 1 ($10x$*), then past the 4 (*$100x$*).*

79. Express the sequence of powers of 10 from 10^1 to 10^5, and multiply the second term by the fourth term. [**Easy**] [**Competency-Based**]
Hint: What is 100×10000*?*

80. Prove that the absolute value equation $|x - 5| = 2$ has two distinct rational solutions. [**Difficult**] [**Competency-Based**]
Hint: Distance from x *to 5 is 2. The solutions are* $5 + 2$ *and* $5 - 2$*.*

SECTION D: 5-Mark Questions (15 Questions)

81. **Geometry and Irrational Numbers:** A town planner is designing a rectangular public square that is exactly 3 km long and 1 km wide. They want to lay a straight fiber-optic cable exactly along the diagonal. (i) Calculate the exact theoretical length of this cable. (ii) Prove, using the method of contradiction, that this specific length cannot be expressed as a ratio of two integers $\frac{p}{q}$. [**HOTS**] [**Competency-Based**]
Hint: Diagonal = $\sqrt{3^2 + 1^2} = \sqrt{10}$*. Adapt the proof to show* p^2 *is a multiple of 10.*

82. **Algorithmic Fractions:** A simulation engine renders a repeating decimal coordinate $x = 4.1\overline{85}$. (i) Convert this coordinate into a precise fraction in the form $\frac{p}{q}$. (ii) If the engine needs to locate a point exactly halfway between x and 5.0, calculate that new coordinate as a fraction. [**Difficult**] [**Competency-Based**]
Hint: Let $x = 4.18585...$ *Multiply by 10, then by 1000. Subtract to get* $\frac{p}{q}$*. Then average with* $\frac{5}{1}$*.*

83. **Number Line Construction:** An automated laser must target a point exactly $\sqrt{5}$ meters from its origin. (i) Describe step-by-step the geometric construction required to accurately mark $\sqrt{5}$ on a physical number line using only a straightedge and compass. (ii) Explain the underlying Pythagorean theorem calculations for your steps. [**HOTS**] [**Competency-Based**]
Hint: Create a right triangle on the number line with base 2 units and height 1 unit.

84. **Compound Rational Operations:** A pharmaceutical lab scales up a chemical batch. The original recipe calls for $\frac{25}{6}$ liters of solution. They first increase the recipe by a factor of $\frac{4}{3}$. Later, they must divide the new total equally among 5 smaller vats. (i) Calculate the exact rational volume of solution in one vat. (ii) Determine if this final volume will yield a terminating or repeating decimal on the digital display, and justify using prime factorization. [**Difficult**] [**Competency-Based**]
Hint: Multiply $\left(\frac{25}{6} \times \frac{4}{3} \times \frac{1}{5}\right)$*. Simplify and check the prime factors of the denominator.*

85. **Density of Rationals in Engineering:** An aeronautical engineer designing a turbine needs exactly 4 distinct rational thickness tolerances strictly between $a = \frac{4}{7}$ mm and $b = \frac{3}{5}$ mm. (i) Express a and b with a sufficiently large common denominator to find these 4 integer-numerator fractions. (ii) State the 4 rational numbers. (iii) Why is the density property of rational numbers essential for continuous physical manufacturing? [**HOTS**] [**Competency-Based**]
Hint: Common denominator 35 yields $\frac{20}{35}$ *and* $\frac{21}{35}$*. Multiply top and bottom by 5 to create a large enough gap between numerators.*

86. **Brahmagupta's Ledger Application:** A multinational corporation audits four regional branches. Branch A net is $+80{,}000$. Branch B is $-40{,}000$. Branch C is $-55{,}000$. Branch D is $+20{,}000$. (i) Calculate the total aggregate value using integer addition. (ii) The CEO decides to "forgive" (subtract) Branch C's debt from the corporate books. Mathematically show how subtracting this

debt affects the total aggregate value, and calculate the new total. (iii) Explain this outcome using Brahmagupta's rule for subtracting a debt. [**Easy**] [**Competency-Based**]
Hint: Add all four. Then subtract $(-55{,}000)$. *Subtracting a negative increases net wealth.*

87. **Infinite Series and π:** A developer programs a robot's turning radius using Madhava's infinite series: $\pi = 4 \times (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots)$. (i) Calculate the fractional approximation of π using just the first four terms inside the bracket. (ii) Convert your fractional result to a decimal (up to 3 decimal places). (iii) Discuss why extending this series infinitely is the only way to perfectly represent π, referring to irrationality. [**Difficult**] [**Competency-Based**]
Hint: Evaluate $4 \times (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7})$ *with common denominator 105.*

88. **Predicting Decimal Behavior:** A cloud server splits data packets into fractions based on active users q, giving each user $\frac{p}{q}$ of the bandwidth. (i) If $q = 800$, will the data allocation form a terminating or repeating decimal? Prove it. (ii) If $q = 150$, prove why the allocation will be a repeating decimal. (iii) State the general rule connecting the prime factorization of q to decimal termination. [**HOTS**] [**Competency-Based**]
Hint: $800 = 2^5 \times 5^2$. $150 = 2 \times 3 \times 5^2$. *Presence of 3 forces a repeating decimal.*

89. **Absolute Value in Navigation:** Two weather drones are measuring atmospheric pressure. Relative to a zero baseline, Drone Alpha is at altitude $x = -\frac{17}{3}$ km and Drone Beta is at $y = -\frac{23}{4}$ km. (i) Find the absolute altitude of each drone. Which one is higher (closer to 0)? (ii) Calculate the exact vertical distance between them using $|x - y|$. (iii) Represent this scenario on a vertical number line. [**Difficult**] [**Competency-Based**]
Hint: Convert to common denominator 12. $-\frac{68}{12}$ *and* $-\frac{69}{12}$.

90. **Properties of Real Numbers:** Evaluate the expression $\left(-\frac{3}{4}\right) \times \left(\frac{8}{5} - \frac{10}{3}\right)$ in two different ways to verify the Distributive Property. (i) Method 1: Solve inside the bracket first, then multiply. (ii) Method 2: Distribute the multiplication first, then subtract. (iii) Compare the results. [**Easy**] [**Competency-Based**]
Hint: Both methods should yield the same positive rational number.

91. **Cyclic Multiplication:** Evaluate the long division for $\frac{1}{13}$. Does it form a cyclic block? Verify by calculating $\frac{2}{13}$ and $\frac{3}{13}$ to see if the block merely shifts or changes completely. [**Difficult**] [**Competency-Based**]
Hint: $\frac{1}{13}$ *yields a 6-digit repeating block* 076923. *Check if multiplying it by 2 or 3 rotates these exact digits.*

92. **Complex Integer Chains:** A trader starts with 0. He takes 3 loans of 500. He makes 4 profits of 300. He pays off 1 loan of 500 (subtracts a debt). Write the single mathematical expression using integers. Solve it to find his net worth. [**HOTS**] [**Competency-Based**]
Hint: $3(-500) + 4(+300) - 1(-500)$.

93. **Fraction to Decimal Logic:** Without actually doing the long division, explain step-by-step how you can determine the exact decimal expansion of $7/25$ by manipulating the fraction into a power of 10. [**Easy**] [**Competency-Based**]
Hint: Multiply numerator and denominator by 4.

94. **Algebraic Rationality Proof:** Prove that if x and y are both rational numbers (where $y \neq 0$), then the expression $\frac{2x+3y}{y}$ is always a rational number. [**Difficult**] [**Competency-Based**]
Hint: Let $x = \frac{a}{b}$ *and* $y = \frac{c}{d}$*. Substitute and show the result is a ratio of integers.*

95. **Scaling Fractions:** A blueprint requires a line segment to be scaled by a factor of $3/7$. If the original line is 2.45 cm, calculate the scaled length exactly as a fraction and a decimal. [**HOTS**] [**Competency-Based**]
Hint: Convert 2.45 *to* $\frac{245}{100}$*, simplify, then multiply by* $\frac{3}{7}$*.*

SECTION E: Case Study Based Questions (Q96 - Q100)

(Strictly formatted as per CBSE guidelines: 1 Mark + 1 Mark + 2 Marks with internal choice).

96. **Case Study 1: Deep Sea Submersibles (Integers & Absolute Value)** [**Easy**]
An oceanography team is tracking their depth relative to Sea Level (0 meters). On Dive 1, they descend 350m. On Dive 2, a strong current pushes them upwards by 120m. On Dive 3, they descend another 280m. The team's sonar maps absolute vertical distance traveled to calculate battery life, while the depth gauge tracks net elevation.

    a. What is the team's net elevation relative to Sea Level at the end of Dive 2? [**1 Mark**]

    b. Calculate the total absolute vertical distance the submersible traveled over all three dives. [**1 Mark**]

    c. Find the final net elevation at the end of Dive 3.

    OR

    If the team decides to return exactly to Sea Level from their Dive 3 position in a single trip, represent this return journey integer. [**2 Marks**]

Hint: Absolute distance adds $|-350| + |+120| + |-280|$.

97. **Case Study 2: Baking and Ratios (Decimals & Fractions)** [**Difficult**]
A culinary app is designing an automated recipe scaler. The system must divide the total volume of dough equally based on the number of pastries. A small batch yields 5 pastries $\left(\frac{1}{5}\right)$, a medium yields 12 pastries $\left(\frac{1}{12}\right)$, and a large yields 16 pastries $\left(\frac{1}{16}\right)$. The developer notices some fractions cause memory errors because their decimal values repeat infinitely.

    a. Which of the three batch sizes will result in a repeating decimal? [**1 Mark**]

    b. State the prime factorization of the denominator for the large batch $\left(\frac{1}{16}\right)$. [**1 Mark**]

    c. Prove why the medium batch $\left(\frac{1}{12}\right)$ results in a non-terminating repeating decimal based on its prime factors.

    OR

Convert the fraction for the small batch $\left(\frac{1}{5}\right)$ into its exact terminating decimal form using the power of 10 method without long division. [**2 Marks**]

Hint: Prime factors must only be 2 or 5 to terminate.

98. **Case Study 3: Urban Planning (Irrational Diagonals)** [**HOTS**]
A city planner is constructing a new rectangular plaza measuring 40 meters by 50 meters. To allow pedestrians to cross quickly, a diagonal walkway is paved directly across the plaza. While ordering materials, the contractor realizes that some diagonal lengths (like a 20×20 meter square park) yield irrational numbers that cannot be perfectly measured.

   a. Calculate the exact length of the diagonal walkway for the 40×50 meter plaza. [**1 Mark**]

   b. Is the diagonal length of this specific 40×50 plaza a rational or irrational number? [**1 Mark**]

   c. Calculate the exact diagonal length of the 20×20 meter square park.

   OR

   Explain mathematically why the diagonal of the 20×20 meter park is considered an irrational number. [**2 Marks**]

Hint: Use $a^2 + b^2 = c^2$. $\sqrt{4100}$ vs $\sqrt{800}$.

99. **Case Study 4: Data Encryption (Cyclic Decimals)** [**Difficult**]
A cybersecurity algorithm uses the unique properties of cyclic numbers to generate encryption keys. They base their algorithm on the fraction $\frac{1}{7}$, which equals $0.\overline{142857}$. To encode a message, the system multiplies this fraction by an integer from 1 to 6, which shifts the sequence of the six digits without changing the digits themselves.

   a. If the system generates the decimal $0.\overline{571428}$, what integer was $\frac{1}{7}$ multiplied by? [**1 Mark**]

   b. How many possible non-zero remainders exist when performing the long division of $1 \div 7$? [**1 Mark**]

   c. Using the cyclic property, determine the exact repeating decimal sequence for $\frac{6}{7}$.

   OR

   Explain algebraically how to convert the pure repeating decimal $0.\overline{142857}$ back into the fraction $\frac{1}{7}$. [**2 Marks**]

Hint: Look at the first digit of the shifted sequence to determine the multiplier.

100. **Case Study 5: Quantum Physics Probabilities (Rational Density)** [**HOTS**]
A physicist is mapping probability amplitudes in a quantum field. She has two stable nodes at amplitudes of $\frac{2}{5}$ and $\frac{3}{7}$. She needs to locate a highly sensitive third node exactly halfway between the two existing nodes to ensure continuous data coverage without interference.

a. Between $\frac{2}{5}$ and $\frac{3}{7}$, which amplitude is mathematically greater? [**1 Mark**]

b. What mathematical property guarantees that an amplitude exactly between these two nodes exists? [**1 Mark**]

c. Calculate the exact rational amplitude where the third node must be located.

OR

Find two distinct rational amplitudes that lie strictly between $\frac{2}{5}$ and $\frac{3}{7}$ by using a common denominator of 70. [**2 Marks**]

Hint: Use the average formula $\frac{a+b}{2}$ *to find the exact halfway point.*

EXPERT ANSWER KEY & STEP-WISE MARKING SCHEME

SECTION A (1-Mark Questions)

(1 mark for each correct answer)

1. B (Above ground is positive, descending is subtracting a positive/adding a negative).
2. B (Terminating decimals only have prime factors of 2 and/or 5 in the denominator).
3. D (Side is $\sqrt{12}$, which is not a perfect square, hence irrational).
4. C (40 divided by 6 leaves a remainder of 4. The 4th digit in 142857 is 8).
5. B (Debt × Fortune = Debt. $-15 \times 20 = -300$).
6. C ($\frac{100}{6}$ simplifies to $\frac{50}{3}$. Denominator has prime factor 3, so it repeats).
7. B (Average: $\frac{3.2+3.3}{2} = 3.25$).
8. B ($0.\overline{9} = 1$).
9. B ($2^2 + 3^2 = 4 + 9 = 13 = (\sqrt{13})^2$).
10. B (Absolute value measures magnitude/distance from zero ignoring sign).
11. B (It is non-terminating and non-repeating because the pattern changes/grows).
12. C (Adding a fortune and an equal debt equals zero).
13. A (Simplify $\frac{21}{14}$ to $\frac{3}{2}$. Denominator is 2, so it terminates).
14. D ($6/7 = 0.\overline{857142}$, starts with 8).
15. C ($\sqrt{5^2 + 12^2} = \sqrt{169} = 13$).
16. B (Maximum non-zero remainders for prime p is $p - 1$. For 19, it's 18).

17. A ($0.625 = \frac{625}{1000} = \frac{5}{8}$).

18. B ($\sqrt{4} = 2$ and $\sqrt{9} = 3$. Only two can be measured precisely).

19. A ($4 \times \left(1 - \frac{1}{3}\right) = 4 \times \left(\frac{2}{3}\right) = \frac{8}{3} = 2.66\ldots$).

20. B (Subtracting a negative removes a debt, mathematically equating to a fortune/increase).

21. Irrational.

22. Dhana (Fortune).

23. $\frac{7}{9}$.

24. Density.

25. Cyclic.

26. False (It is a rational approximation, not an exact irrational value).

27. True ($|-850| = |+850| = 850$).

28. False ($10/3$ results in a repeating decimal $3.333\ldots$).

29. True (Hippasus used contradiction to prove $\sqrt{2}$ is irrational).

30. True ($(-4) \times (-20) = +80$).

31. (a) (A is true, R is true and explains A).

32. (d) (A is false: $a - 0 = a$, not 0; R is true).

33. (a) (A is true, R is true and explains A).

34. (d) (A is false, they are not the exact same location; R is true).

35. (a) (A is true, R is true and explains A. $(-18) + (-14) = -32$).

36. (a) (A is true, R is true and explains A. $\sqrt{9} = 3$).

37. (d) (A is false, you can always find a midpoint; R is true).

38. (a) (A is true, R is true and explains A).

39. (a) (A is true, R is true and explains A. $|-30| = 30 > |-5| = 5$).

40. (a) (A is true, R is true and explains A. $0.\overline{9} = 1.0$).

SECTION B (2-Mark Questions)

41. Equation: $-25 - 18 + 12$ **[1 Mark]**
Final depth: $-43 + 12 = -31$ meters. **[1 Mark]**

42. Decimals to fractions: $0.4 = \frac{2}{5} = \frac{16}{40}$ and $0.45 = \frac{9}{20} = \frac{18}{40}$. **[1 Mark]**

Halfway (average): $\frac{16/40+18/40}{2} = \frac{34/40}{2} = \frac{17}{40}$ cm. **[1 Mark]**

43. Simplify radical: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$. **[1 Mark]**

Since $\sqrt{3}$ is irrational, $2\sqrt{3}$ is irrational. **[1 Mark]**

44. Setup fraction: $\frac{145}{1000}$. **[1 Mark]**

Simplify by dividing by 5: $\frac{29}{200}$ grams. **[1 Mark]**

45. Wire length represents physical distance, which cannot be negative. **[1 Mark]**

$|+4-(-1)| = |+5| = 5$ meters. **[1 Mark]**

46. Prime factorization of $40 = 2^3 \times 5$. **[1 Mark]**

Because it contains exclusively prime factors 2 and 5, it is a terminating decimal. **[1 Mark]**

47. Integer multiplication: $(-4) \times (-300) = +1200$. **[1 Mark]**

Removing debts results in a net gain (Fortune) of 1200. **[1 Mark]**

48. Let $x = 0.4545\ldots \Longrightarrow 100x = 45.4545\ldots$. **[1 Mark]**

Subtract: $99x = 45 \Longrightarrow x = \frac{45}{99} = \frac{5}{11}$. **[1 Mark]**

49. Formula: $|5.2 - (-1.8)|$ **[1 Mark]**

Evaluation: $|5.2 + 1.8| = |7.0| = 7$ million spread. **[1 Mark]**

50. $\frac{3}{7}$ starts with third lowest digit (4): $0.\overline{428571}$. **[1 Mark]**

$\frac{4}{7}$ starts with fourth lowest digit (5): $0.\overline{571428}$. **[1 Mark]**

51. Assume $1.5 \times \sqrt{2} = \frac{p}{q}$ (rational). Then $\frac{3}{2}\sqrt{2} = \frac{p}{q} \Longrightarrow \sqrt{2} = \frac{2p}{3q}$. **[1 Mark]**

RHS is rational, but LHS ($\sqrt{2}$) is irrational. Contradiction. Product is irrational. **[1 Mark]**

52. Improper fractions: $\frac{10}{3}$ and $\frac{9}{4}$. **[1 Mark]**

Common denominator 12: $\frac{40}{12} + \frac{27}{12} = \frac{67}{12}$ cups. **[1 Mark]**

53. Multiply numerators and denominators: $\frac{5\times-4}{12\times15} = \frac{-20}{180}$. **[1 Mark]**

Simplify: $-\frac{1}{9}$. **[1 Mark]**

54. Let $x = 0.611\ldots \Longrightarrow 10x = 6.11\ldots \Longrightarrow 100x = 61.11\ldots$. **[1 Mark]**

Subtract: $90x = 55 \Longrightarrow x = \frac{55}{90} = \frac{11}{18}$ meters. **[1 Mark]**

55. LHS: $500 - (-45) = 545$. **[1 Mark]**

RHS: $500 + 45 = 545$. Subtracting a debt equals adding a fortune. **[1 Mark]**

56. Multiply numerator and denominator by 5: $\frac{13\times5}{20\times5}$. **[1 Mark]**

Result: $\frac{65}{100} = 0.65$ km. **[1 Mark]**

57. Placeholder merely separates values (e.g., distinguishing 11 from 101). **[1 Mark]**

Operational zero allows dynamic calculation (e.g., multiplying variable by 0 zeroes out memory, $a \times 0 = 0$). **[1 Mark]**

58. Evaluate absolute values: $8 + 12 - 5$. **[1 Mark]**

Calculate: $20 - 5 = 15$. **[1 Mark]**

59. Cross-multiply: $(-3) \times (-280) = 840$. **[1 Mark]**

$8 \times 105 = 840$. Products are equal, so fractions are equivalent. **[1 Mark]**

60. Setup multiplication: $\frac{5}{2} \times \frac{1}{3}$. **[1 Mark]**

Result: $\frac{5}{6}$ cups. **[1 Mark]**

SECTION C (3-Mark Questions)

61. (i) Equation: $(+350) + (-180) + (-220)$. **[1 Mark]**

(ii) Net change: $350 - 400 = -50$ units. **[1 Mark]**

(iii) The final state is a "debt" (deficit/reduction). **[1 Mark]**

62. Let $x = 2.31515...$ Multiply by 10: $10x = 23.1515...$ **[1 Mark]**

Multiply by 1000: $1000x = 2315.1515...$ **[1 Mark]**

Subtract: $990x = 2292 \Longrightarrow x = \frac{2292}{990} = \frac{382}{165}$. **[1 Mark]**

63. (i) Halfway Alpha & Beta: $\frac{-4.5+1.5}{2} = \frac{-3.0}{2} = -1.5$ km. **[1.5 Marks]**

(ii) Halfway Beta & Gamma: $\frac{1.5+6.5}{2} = \frac{8.0}{2} = 4.0$ km. **[1.5 Marks]**

64. Assume $\sqrt{7} = \frac{p}{q}$ (co-prime). Square both sides: $7 = \frac{p^2}{q^2} \Longrightarrow 7q^2 = p^2$. **[1 Mark]**

p^2 is a multiple of 7, so p must be a multiple of 7 ($p = 7k$). **[1 Mark]**

Substitute: $7q^2 = 49k^2 \Longrightarrow q^2 = 7k^2$. So q is also a multiple of 7. Both share factor 7. Contradiction. **[1 Mark]**

65. (i) $\frac{1}{10}$ (Standard) terminates. $\frac{1}{15}$ (Giant) repeats. **[1 Mark]**

(ii) Prime factorization of 10 is 2×5. It only has 2 and 5, so it terminates. **[1 Mark]**

(iii) Prime factorization of 15 is 3×5. The presence of 3 makes it repeat. **[1 Mark]**

66. Improper fractions: $\frac{81}{4}, -\frac{17}{2}, \frac{43}{8}$. **[1 Mark]**

Common denominator 8: $\frac{162}{8} - \frac{68}{8} + \frac{43}{8}$. **[1 Mark]**

Result: $\frac{162-68+43}{8} = \frac{137}{8} = 17\frac{1}{8}$ m. **[1 Mark]**

67. (i) $x = 0.\overline{6} = \frac{6}{9} = \frac{2}{3}$. $z = 0.08\overline{3} \Longrightarrow 900z = 75 \Longrightarrow z = \frac{75}{900} = \frac{1}{12}$. **[1.5 Marks]**

(ii) $y = 1/4$. Sum $= \frac{2}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12} + \frac{3}{12} + \frac{1}{12} = \frac{12}{12} = 1$. Yes. **[1.5 Marks]**

68. Substitution: $C = \frac{5}{9}(77 - 32)$. **[1 Mark]**

Bracket subtraction: $C = \frac{5}{9}(45)$. **[1 Mark]**

Multiplication/Simplification: $C = 5 \times 5 = 25°\text{C}$. **[1 Mark]**

69. Setup cross-multiplication for equivalence $\frac{1017}{3051} = \frac{1}{3}$. **[1 Mark]**

Calculate LHS: $1017 \times 3 = 3051$. **[1 Mark]**

Calculate RHS: $3051 \times 1 = 3051$. Since $ad = bc$, they are equivalent. **[1 Mark]**

70. Evaluate brackets (B+C): $(+900) + (-1500) = -600$ (Debt). **[1 Mark]**

Multiply $A \times (B + C)$: $(-400) \times (-600)$. **[1 Mark]**

Result: $+240{,}000$. A debt times a debt is a Fortune. **[1 Mark]**

71. $\frac{1}{7}$ cycle is 1-4-2-8-5-7. **[1 Mark]**

$\frac{2}{7}$ starts with 2nd lowest digit (2): $0.\overline{285714}$. **[1 Mark]**

$\frac{5}{7}$ starts with 5th lowest digit (7): $0.\overline{714285}$. **[1 Mark]**

72. Distribute: $\left(-\frac{4}{7} \times \frac{14}{3}\right) + \left(-\frac{4}{7} \times \frac{21}{8}\right)$. **[1 Mark]**

Multiply: $\left(-\frac{56}{21}\right) + \left(-\frac{84}{56}\right) = \left(-\frac{8}{3}\right) + \left(-\frac{3}{2}\right)$. **[1 Mark]**

Add with common denom 6: $-\frac{16}{6} - \frac{9}{6} = -\frac{25}{6}$. **[1 Mark]**

73. Equation: $\left(+\frac{3}{4}\right) + \left(-\frac{2}{5}\right)$. **[1 Mark]**

Common denominator 20: $\frac{15}{20} - \frac{8}{20} = +\frac{7}{20}$ liters/sec net gain. **[2 Marks]**

74. Algebraic proof: Let $x = 0.999\ldots \Longrightarrow 10x = 9.999\ldots \Longrightarrow 9x = 9 \Longrightarrow x = 1$. **[1.5 Marks]**

Practical Application: $0.\overline{9}$ seconds is mathematically identical to 1 second; the clock measures the exact same physical duration. **[1.5 Marks]**

75. Decimal calculation: $3927 \div 1250 = 3.1416$. **[1 Mark]**

Does it prove π is rational? No. **[1 Mark]**

1Reason: It is only a terminating approximation (*asanna*). π itself never terminates or repeats. **[1 Mark]**

76. Multiplication first: $(-25) \times (-4) = +100$. **[1.5 Marks]**

Division: $(+100) \div (-10) = -10$. **[1.5 Marks]**

77. Common denominator 15: $-\frac{10}{15}$ and $\frac{3}{15}$. **[1 Mark]**

Any three distinct valid fractions: $-\frac{9}{15}, \frac{0}{15}, \frac{2}{15}$, etc. **[2 Marks]**

78. Let $x = 2.1444\ldots$. Multiply by 10: $10x = 21.444\ldots$. **[1 Mark]**

Multiply by 100: $100x = 214.444\ldots$. **[1 Mark]**

Subtract: $90x = 193 \Longrightarrow x = \frac{193}{90}$. **[1 Mark]**

79. Sequence: $10, 100, 1000, 10000, 100000$. **[1.5 Marks]**

Product of 2nd and 4th: $100 \times 10000 = 1{,}000{,}000$ (or 10^6). **[1.5 Marks]**

80. $|x - 5| = 2$ means distance from x to 5 is 2 units. **[1 Mark]**

Case 1: $x - 5 = 2 \Longrightarrow x = 7$. **[1 Mark]**

Case 2: $x - 5 = -2 \Longrightarrow x = 3$. Both 7 and 3 are rational. **[1 Mark]**

SECTION D (5-Mark Questions)

81. **Geometry & Irrational Numbers**

(i) By Pythagorean theorem: Diagonal $c = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$ km. **[1 Mark]**

(ii) Assume $\sqrt{10} = \frac{p}{q}$ in simplest form. Square: $10 = \frac{p^2}{q^2} \Longrightarrow 10q^2 = p^2$. **[1 Mark]**

p^2 is a multiple of 10, so p must be a multiple of 10. Let $p = 10k$. **[1 Mark]**

Substitute: $10q^2 = (10k)^2 = 100k^2 \Longrightarrow q^2 = 10k^2$. **[1 Mark]**

q^2 is a multiple of 10, so q is a multiple of 10. Both p and q share factor 10. Contradiction. Path length $\sqrt{10}$ is irrational. **[1 Mark]**

82. **Algorithmic Fractions**

(i) Let $x = 4.18585... \Longrightarrow 10x = 41.8585...$ **[1 Mark]**

Multiply original by 1000: $1000x = 4185.8585...$ **[1 Mark]**

Subtract: $990x = 4144 \Longrightarrow x = \frac{4144}{990} = \frac{2072}{495}$. **[1 Mark]**

(ii) Average formula: $\frac{\frac{4144}{990}+\frac{5}{1}}{2}$. **[1 Mark]**

Common denominator 990: $\frac{\frac{4144}{990}+\frac{4950}{990}}{2} = \frac{\frac{9094}{990}}{2} = \frac{9094}{1980} = \frac{4547}{990}$. **[1 Mark]**

83. **Number Line Construction**

(i) Step 1: Draw a number line. At coordinate 2, erect a perpendicular of exactly 1 unit length. **[1.5 Marks]**

Step 2: Connect the origin (0) to the top of this perpendicular to form the hypotenuse. **[1.5 Marks]**

Step 3: Use a compass (point at 0) to swing this hypotenuse down to intersect the number line at $\sqrt{5}$. **[0.5 Marks]**

(ii) Pythagorean Theorem: Base $a = 2$, Height $b = 1$. $c^2 = 2^2 + 1^2 = 4 + 1 = 5 \Longrightarrow c = \sqrt{5}$. **[1.5 Marks]**

84. **Compound Rational Operations**

(i) Increase batch: $\frac{25}{6} \times \frac{4}{3} = \frac{100}{18}$ L. **[1.5 Marks]**

Divide by 5: $\frac{100}{18} \times \frac{1}{5} = \frac{20}{18} = \frac{10}{9}$ L. **[1.5 Marks]**

(ii) Decimal behavior: Look at denominator 9. Prime factorization is 3^2. **[1 Mark]**

Since it contains prime factor 3 (not just 2 or 5), it will be a repeating decimal. **[1 Mark]**

85. **Density of Rationals in Engineering**

(i) Original fractions: $\frac{4}{7}$ and $\frac{3}{5}$. Common denominator 35 yields $\frac{20}{35}$ and $\frac{21}{35}$. **[1 Mark]**

We need 4 numbers, so gap $> 4 + 1$. Multiply by 5: $\frac{100}{175}$ and $\frac{105}{175}$. **[1 Mark]**

(ii) 4 distinct rational numbers: $\frac{101}{175}, \frac{102}{175}, \frac{103}{175}, \frac{104}{175}$. **[2 Marks]**

(iii) It guarantees that engineers can safely specify an exact, infinitely precise halfway measurement between any two tolerances to fit parts. **[1 Mark]**

86. **Brahmagupta's Ledger Application**

(i) Total: $(+80000) + (-40000) + (-55000) + (+20000) = 100000 - 95000 = +5000$. **[2 Marks]**

(ii) Subtract debt C: $+5000 - (-55000) = 5000 + 55000 = +60000$. **[1.5 Marks]**

(iii) Brahmagupta's rule: Subtracting a debt represents removing a financial burden, practically equivalent to adding a fortune. **[1.5 Marks]**

87. **Infinite Series and π**

(i) Inside bracket: $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$. Common denominator 105: $\frac{105}{105} - \frac{35}{105} + \frac{21}{105} - \frac{15}{105} = \frac{76}{105}$. **[1.5 Marks]**

Multiply by 4: $4 \times \left(\frac{76}{105}\right) = \frac{304}{105}$. **[1 Mark]**

(ii) Decimal conversion: $304 \div 105 = 2.895\ldots$. **[1.5 Marks]**

(iii) π is irrational, meaning its true value cannot be written as a finite fraction. Only an infinite non-repeating sum can express it. **[1 Mark]**

88. **Predicting Decimal Behavior**

(i) $q = 800$. Prime factors: $800 = 2^5 \times 5^2$. **[1 Mark]**

Exclusively 2 and 5, so it will terminate. **[1 Mark]**

(ii) $q = 150$. Prime factors: $150 = 2 \times 3 \times 5^2$. **[1 Mark]**

Contains factor 3, so it repeats. **[1 Mark]**

(iii) Rule: A fraction $\frac{p}{q}$ (in simplest form) terminates iff q has only prime factors of 2 and 5. Otherwise, it repeats. **[1 Mark]**

89. **Absolute Value in Navigation**

(i) Absolute altitudes: $\left|-\frac{17}{3}\right| = \frac{17}{3} = 5.66$ km. $\left|-\frac{23}{4}\right| = \frac{23}{4} = 5.75$ km. **[1 Mark]**

Alpha (5.66 km) is higher (closer to 0). **[1 Mark]**

(ii) Distance: $\left|\left(-\frac{17}{3}\right) - \left(-\frac{23}{4}\right)\right|$. Common denominator 12: $\left|\left(-\frac{68}{12}\right) + \left(\frac{69}{12}\right)\right|$. **[1 Mark]**

Distance $= \left|\frac{1}{12}\right| = \frac{1}{12}$ km. **[1 Mark]**

(iii) Number line: 0 at top, Alpha above Beta. Gap is exactly $\frac{1}{12}$ km. **[1 Mark]**

90. **Properties of Real Numbers**

(i) Method 1 (Bracket first): $\frac{8}{5} - \frac{10}{3} = \frac{24}{15} - \frac{50}{15} = -\frac{26}{15}$. **[1 Mark]**

Multiply: $\left(-\frac{3}{4}\right) \times \left(-\frac{26}{15}\right) = +\frac{78}{60} = \frac{13}{10}$. **[1 Mark]**

(ii) Method 2 (Distribute): $\left(-\frac{3}{4} \times \frac{8}{5}\right) - \left(-\frac{3}{4} \times \frac{10}{3}\right)$. **[1 Mark]**

$\left(-\frac{24}{20}\right) - \left(-\frac{30}{12}\right) = \left(-\frac{6}{5}\right) - \left(-\frac{5}{2}\right) = \left(-\frac{12}{10}\right) + \left(\frac{25}{10}\right) = \frac{13}{10}$. **[1 Mark]**

(iii) Both methods yield $\frac{13}{10}$, perfectly verifying the Distributive Property. **[1 Mark]**

91. **Cyclic Multiplication**

Long division $1 \div 13 = 0.\overline{076923}$. **[1.5 Marks]**

$\frac{2}{13} = 0.\overline{153846}$. **[1.5 Marks]**

$\frac{3}{13} = 0.\overline{230769}$. **[1 Mark]**

The digits change completely; it does not simply shift the same 6 digits. (It has multiple cyclic groups). **[1 Mark]**

92. **Complex Integer Chains**

Expression: $3(-500) + 4(+300) - 1(-500)$. **[2 Marks]**

Multiplication: $-1500 + 1200 + 500$. **[1.5 Marks]**

Addition: $-300 + 500 = +200$. Net worth is 200 (Fortune). **[1.5 Marks]**

93. **Fraction to Decimal Logic**

Step 1: Identify that the denominator 25 is 5^2. **[1.5 Marks]**

Step 2: To make it a power of 10, multiply denominator by $2^2 = 4$ to get 100. **[1.5 Marks]**

Step 3: Multiply numerator by 4 as well: $7 \times 4 = 28$. **[1 Mark]**

Result: $\frac{28}{100} = 0.28$. **[1 Mark]**

94. **Algebraic Rationality Proof**

Let $x = \frac{a}{b}$ and $y = \frac{c}{d}$ (where a, b, c, d are integers, $b, c, d \neq 0$). **[1 Mark]**

Substitute: $\frac{2\left(\frac{a}{b}\right)+3\left(\frac{c}{d}\right)}{\frac{c}{d}}$. **[1 Mark]**

Numerator: $\frac{2ad+3bc}{bd}$. **[1.5 Marks]**

Divide by y: $\left(\frac{2ad+3bc}{bd}\right) \times \left(\frac{d}{c}\right) = \frac{2ad+3bc}{bc}$. **[1 Mark]**

Since integers are closed under addition/multiplication, the result is an integer ratio $\frac{P}{Q}$, thus rational. **[0.5 Marks]**

95. **Scaling Fractions**

Convert decimal: $2.45 = \frac{245}{100} = \frac{49}{20}$. **[2 Marks]**

Scale: $\frac{49}{20} \times \frac{3}{7}$. **[1 Mark]**

Simplify: $\frac{7}{20} \times \frac{3}{1} = \frac{21}{20}$. **[1 Mark]**

Decimal: $\frac{21}{20} = \frac{105}{100} = 1.05$ cm. **[1 Mark]**

SECTION E (Case Study Based Questions)

96. **Case Study 1: Deep Sea Submersibles**

(i) Net elevation Dive 2: $(-350) + (+120) = -230$ meters. **[1 Mark]**

(ii) Absolute distance: $|-350| + |+120| + |-280| = 350 + 120 + 280 = 750$ meters. **[1 Mark]**

(iii) **Choice 1:** Final elevation Dive 3: $-230 - 280 = -510$ meters. **[2 Marks]**
OR

Choice 2: Return journey integer: To get from -510 back to 0, they ascend 510m, which is $+510$. **[2 Marks]**

97. **Case Study 2: Baking and Ratios**

(i) The medium batch $\frac{1}{12}$ results in a repeating decimal. **[1 Mark]**

(ii) Prime factorization of 16 is $2 \times 2 \times 2 \times 2$ (or 2^4). **[1 Mark]**

(iii) **Choice 1:** Denominator 12 factors into $2^2 \times 3$. Because it contains prime factor 3 (other than 2 or 5), the decimal will repeat. **[1 Mark for factors, 1 Mark for reasoning = 2 Marks]**
OR

Choice 2: $\frac{1}{5}$. Multiply num and denom by 2 to get power of 10: $\frac{1\times2}{5\times2} = \frac{2}{10} = 0.2$. **[1 Mark for setup, 1 Mark for 0.2 = 2 Marks]**

98. **Case Study 3: Urban Planning**

(i) Diagonal $c = \sqrt{40^2 + 50^2} = \sqrt{1600 + 2500} = \sqrt{4100} = 10\sqrt{41}$ meters. **[1 Mark]**

(ii) Irrational number (4100 is not a perfect square). **[1 Mark]**

(iii) **Choice 1:** Diagonal $= \sqrt{20^2 + 20^2} = \sqrt{400 + 400} = \sqrt{800} = 20\sqrt{2}$ meters. **[2 Marks]**
OR

Choice 2: The length $\sqrt{800}$ simplifies to $20\sqrt{2}$. Since $\sqrt{2}$ cannot be expressed as a ratio of integers $\frac{p}{q}$, any non-zero rational multiple is also irrational. **[2 Marks]**

99. **Case Study 4: Data Encryption**

(i) $0.\overline{571428}$ starts with the 4th lowest digit (5), so $\frac{1}{7}$ was multiplied by 4. **[1 Mark]**

(ii) 6 possible non-zero remainders (1, 2, 3, 4, 5, 6). **[1 Mark]**

(iii) **Choice 1:** For $\frac{6}{7}$, find the largest digit in 142857 (8). Sequence is $0.\overline{857142}$. **[2 Marks]**
OR

Choice 2: Let $x = 0.\overline{142857}$. Multiply by 1,000,000: $1000000x = 142857.\overline{142857}$. Subtract x: $999999x = 142857 \Longrightarrow x = \frac{142857}{999999} = \frac{1}{7}$. **[2 Marks]**

100. **Case Study 5: Quantum Physics Probabilities**

(i) $\frac{3}{7}$ is greater. ($\frac{2}{5} = 0.400, \frac{3}{7} \approx 0.428$). **[1 Mark]**

(ii) The density property of rational numbers. **[1 Mark]**

(iii) **Choice 1:** Halfway point $= \frac{\frac{2}{5}+\frac{3}{7}}{2}$. Common denom 35: $\frac{\frac{14}{35}+\frac{15}{35}}{2} = \frac{\frac{29}{35}}{2} = \frac{29}{70}$. **[1 Mark for sum, 1 Mark for halving = 2 Marks]**
OR

Choice 2: $\frac{2}{5} = \frac{28}{70}$. $\frac{3}{7} = \frac{30}{70}$. A rational strictly between them is $\frac{29}{70}$. To get two, use 140: $\frac{56}{140}$ and $\frac{60}{140}$. Distinct values: $\frac{57}{140}, \frac{58}{140}, \frac{59}{140}$. **[2 Marks]**