Triangles
Ex. 7.1

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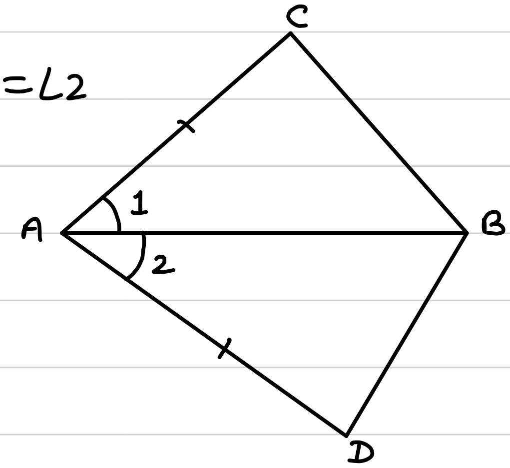


1. **Given** – In □ABCD, $AC = AD$, $\angle 1 = \angle 2$

To prove – $\triangle ABC \cong \triangle ABD$

Proof – In $\triangle ABC$ and $\triangle ABD$

$AC = AD$ (given)

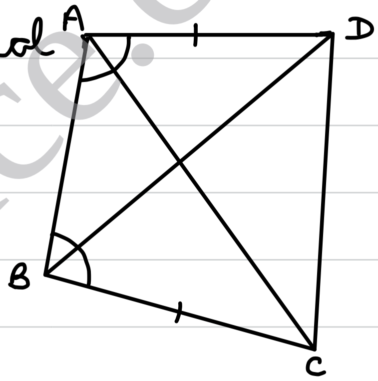
$\angle 1 = \angle 2$ (given)

$AB = AB$ (common)

$\therefore \triangle ABC \cong \triangle ABD$ by SAS congruence

$\therefore BC = BD$ (c.p.c.t)

Hence proved.



2. **Given** – ABCD is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$.

To prove – (i) $\triangle ABD \cong \triangle BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$

Proof – (i) In $\triangle ABD$ and $\triangle BAC$

$AD = BC$ (given)

$\angle DAB = \angle CBA$ (given)

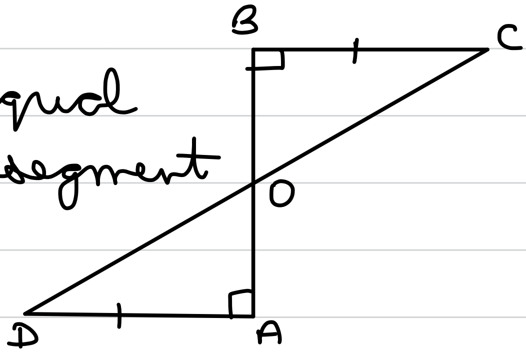
$AB = BA$ (common)

$\therefore \triangle ABD \cong \triangle BAC$ by SAS congruence

(ii) $BD = AC$ (c.p.c.t)

(iii) $\angle ABD = \angle BAC$ (c.p.c.t)

Hence proved.



3. **Given** – AD and BC are equal perpendiculars to line segment AB.

To prove – $OA = OB$

Proof – In $\triangle AOD$ and $\triangle BOC$

$\angle AOD = \angle BOC$ (vertically opposite angles)

$\angle A = \angle B$ (each $= 90°$)

$AD = BC$ (given)

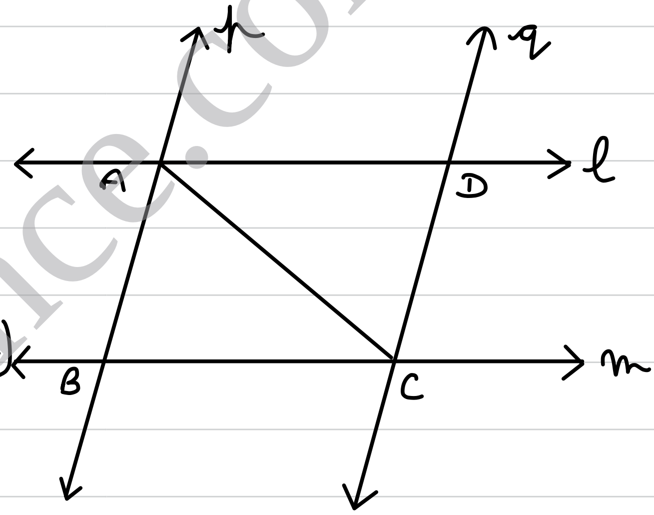
$\therefore \triangle AOD \cong \triangle BOC$ by AAS corollary

$\therefore OA = OB$ (c.p.c.t.)

$\therefore CD$ bisects AB.

Hence proved.

4. **Given** – $l \parallel m$ and $p \parallel q$.



To prove – $\triangle ABC \cong \triangle CDA$

Proof – In $\triangle ABC$ and $\triangle CDA$

$\angle ACB = \angle CAD$ (alternate interior angles)

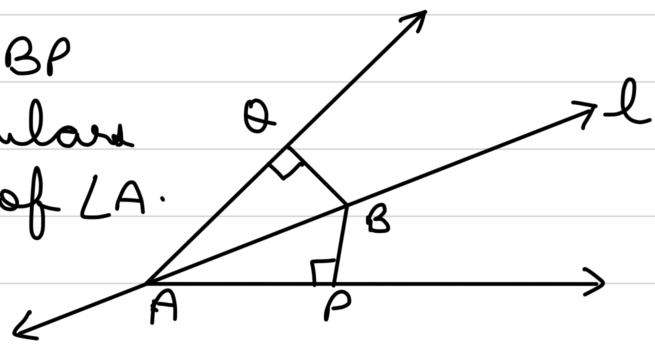
$AC = CA$ (common)

$\angle CAB = \angle ACD$ (alternate interior angles)

$\therefore \triangle ABC \cong \triangle CDA$ by ASA congruence

Hence proved.

5. **Given** – $\angle QAB = \angle PAB$ and BP and BQ are perpendiculars from B to the arms of $\angle A$.



To prove –

(i) $\triangle APB \cong \triangle AQB$

(ii) $BP = BQ$

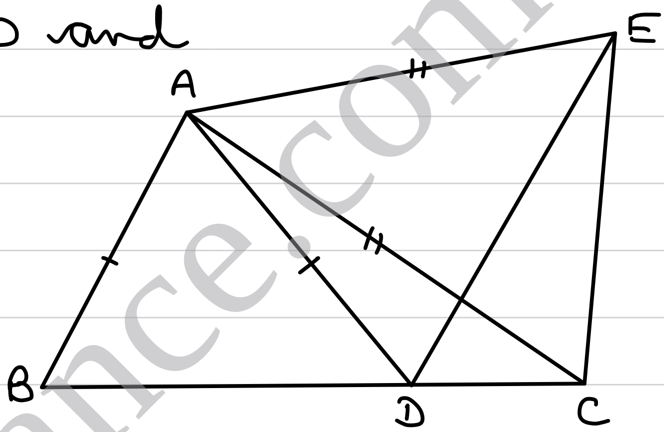
Proof –

(i) In $\triangle APB$ and $\triangle AQB$

$\angle BQA = \angle BPA$ (each $= 90°$)

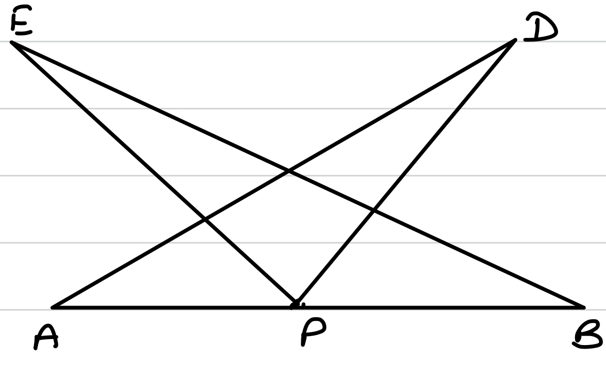
$\angle PAB = \angle QAB$ (given)
$AB = AB$ (common)
$\therefore \triangle APB \cong \triangle AQB$ by AAS corollary
(ii) $\therefore$ $BP = BQ$ (c.p.c.t)
or B is equidistant from the arms of $\angle A$.
Hence proved.

6. <u>Given</u> – $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$.
<u>To prove</u> – $BC = DE$
<u>Proof</u> –



$\angle BAD = \angle EAC$ (given)
Adding $\angle DAC$ on both sides
$\angle BAD + \angle DAC = \angle EAC + \angle DAC$
or $\angle BAC = \angle DAE$ — (1)
In $\triangle BAC$ and $\triangle DAE$
$AB = AD$ (given)
$\angle BAC = \angle DAE$ (using equation (1))
$AC = AE$ (given)
$\therefore \triangle BAC \cong \triangle DAE$ by SAS congruence
$\therefore$ $BC = DE$ (c.p.c.t)
Hence proved.

7. <u>Given</u> – AB is a line segment and $AP = BP$, $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$.
<u>To prove</u> –



(i) $\Delta DAP \cong \Delta EBP$

(ii) $AD = BE$

Proof – $\angle EPA = \angle DPB$ (given)

Adding $\angle EPD$ on both sides

$\angle EPA + \angle EPD = \angle DPB + \angle EPD$

or $\angle DPA = \angle EPB$ — (1)

(i) In ΔDAP and ΔEBP

$\angle A = \angle B$ (given)

$AP = BP$ (given)

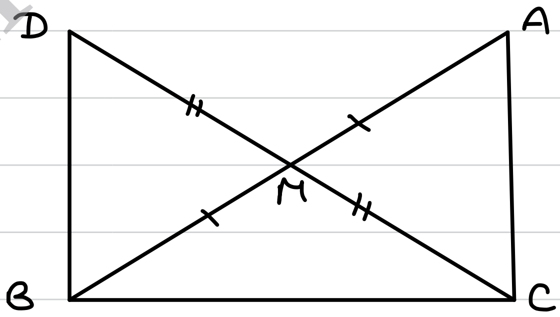
$\angle DPA = \angle EPB$ (using eq^n. (1))

$\therefore \Delta DAP \cong \Delta EBP$ by ASA congruence

(ii) $AD = BE$ (c.p.c.t)

Hence proved.

8. Given – In ΔABC, $\angle ACB = 90°$, $AM = BM$. Also, $DM = CM$



To prove –

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle or $\angle DBC = 90°$

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Proof – (i) In ΔAMC and ΔBMD

$AM = BM$ (given)

$\angle AMC = \angle BMD$ (vertically opposite angles)

$CM = DM$ (given)

$\therefore \Delta AMC \cong \Delta BMD$ by SAS congruence

$\therefore \angle MAC = \angle MBD$ (c.p.c.t) and $AC = DB$ (c.p.c.t) — (1)

But these are alternate interior angles.

$\therefore DB \parallel AC$

(ii) $DB \parallel AC$ (proved) and BC is a transversal

$\therefore \angle DBC + \angle ACB = 180°$ (cointerior angles)

or $\angle DBC + 90° = 180°$ ($\because \angle ACB = 90°$ given)

or $\angle DBC = 180° - 90°$

or $\angle DBC = 90°$

$\therefore \angle DBC$ is a right angle.

(iii) In $\triangle DBC$ and $\triangle ACB$

$DB = AC$ (using eq^n. (1))

$\angle DBC = \angle ACB$ (each $= 90°$)

$BC = CB$ (common)

$\therefore \triangle DBC \cong \triangle ACB$ by SAS congruence

$\therefore DC = AB$ (c.p.c.t)

(iv) $DM + CM = AB$

or $CM + CM = AB$ ($\because DM = CM$ given)

or $2CM = AB$

or $CM = \frac{1}{2} AB$

Hence proved.