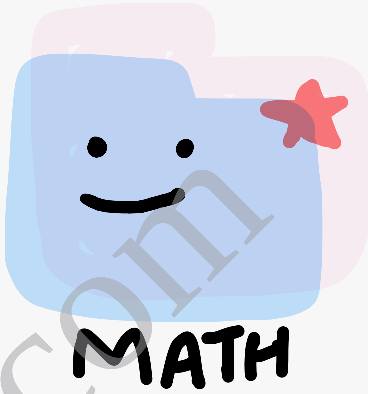
Introduction To Trigonometry Ex. 8.1



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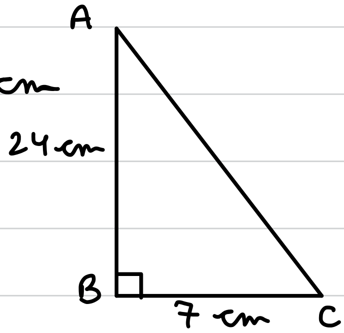
Ex. 8.1



1. In $\triangle ABC$, $\angle B = 90°$, $AB = 24$ cm, $BC = 7$ cm

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or $AC^2 = 24^2 + 7^2$

or $AC^2 = 576 + 49$

or $AC^2 = 625$

or $AC = \sqrt{625}$

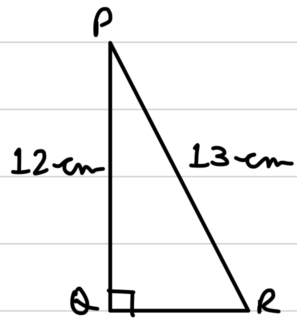
or $AC = 25$ cm

(i) $\sin A = \dfrac{BC}{AC} = \dfrac{7}{25}$

$\cos A = \dfrac{AB}{AC} = \dfrac{24}{25}$

(ii) $\sin C = \dfrac{AB}{AC} = \dfrac{24}{25}$

$\cos C = \dfrac{BC}{AC} = \dfrac{7}{25}$



2. In $\triangle PQR$, $\angle Q = 90°$, $PQ = 12$ cm, $PR = 13$ cm

Using Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

or $13^2 = 12^2 + QR^2$

or $169 = 144 + QR^2$

or $169 - 144 = QR^2$

or $25 = QR^2$

or $\sqrt{25} = QR$

or $QR = 5$ cm

$\tan P = \dfrac{QR}{PQ} = \dfrac{5}{12}$

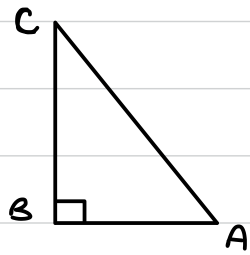
$\cot R = \dfrac{QR}{PQ} = \dfrac{5}{12}$

$\tan P - \cot R$

$= \dfrac{5}{12} - \dfrac{5}{12}$

$= 0$

3. $\sin A = \frac{3}{4}$



or $\frac{BC}{AC} = \frac{3}{4}$

Let $BC = 3k$

$AC = 4k$

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or $(4k)^2 = AB^2 + (3k)^2$

or $16k^2 = AB^2 + 9k^2$

or $16k^2 - 9k^2 = AB^2$

or $7k^2 = AB^2$

or $AB = \sqrt{7}\ k$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

4. $15 \cot A = 8$

or $\cot A = \frac{8}{15}$

or $\frac{AB}{BC} = \frac{8}{15}$

Let $AB = 8k$, $BC = 15k$

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or $AC^2 = (8k)^2 + (15k)^2$

or $AC^2 = 64k^2 + 225k^2$

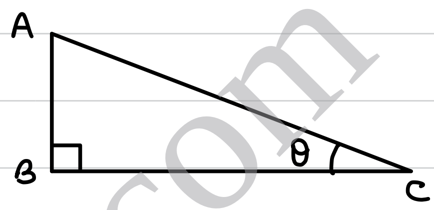
or $AC^2 = 289k^2$

or $AC = \sqrt{289k^2}$

or $AC = 17k$

$\sin A = \dfrac{BC}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$

$\sec A = \dfrac{AC}{AB} = \dfrac{17k}{8k} = \dfrac{17}{8}$



5. $\sec\theta = \dfrac{13}{12}$

or $\dfrac{AC}{BC} = \dfrac{13}{12}$

Let $AC = 13k$, $BC = 12k$

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or $(13k)^2 = AB^2 + (12k)^2$

or $169k^2 = AB^2 + 144k^2$

or $169k^2 - 144k^2 = AB^2$

or $25k^2 = AB^2$

or $\sqrt{25k^2} = AB^2$

or $5k = AB$

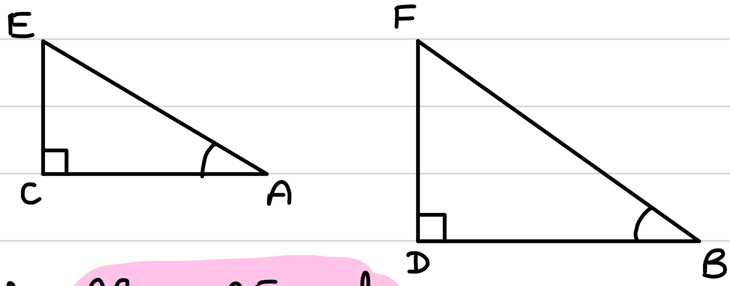
$\sin\theta = \dfrac{AB}{AC} = \dfrac{5k}{13k} = \dfrac{5}{13}$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{12k}{13k} = \dfrac{12}{13}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{5k}{12k} = \dfrac{5}{12}$

$\operatorname{cosec}\theta = \dfrac{AC}{AB} = \dfrac{13k}{5k} = \dfrac{13}{5}$

$\cot\theta = \dfrac{BC}{AC} = \dfrac{12k}{5k} = \dfrac{12}{5}$



6. $\cos A = \cos B$ (given)

or $\dfrac{AC}{AE} = \dfrac{BD}{BF}$

Let $\dfrac{AC}{AE} = \dfrac{BD}{BF} = k$ or $\dfrac{AC}{BD} = \dfrac{AE}{BF} = k$

$\therefore AC = k\,BD$ — (i)

$AE = k\,BF$ — (ii)

Using Pythagoras theorem

In $\triangle ACE$, $CE^2 = AE^2 - AC^2$

or $CE^2 = (k\,BF)^2 - (k\,BD)^2$ (using eq. (i))

or $CE^2 = k^2\,BF^2 - k^2\,BD^2$

or $CE^2 = k^2(BF^2 - BD^2)$ — (iii)

In $\triangle BDF$, $DF^2 = BF^2 - BD^2$ — (iv)

Dividing eq. (iii) by (iv)

$$\frac{CE^2}{DF^2} = \frac{k^2(BF^2 - BD^2)}{BF^2 - BD^2}$$

or $\dfrac{CE^2}{DF^2} = k^2$

Taking square root on both sides

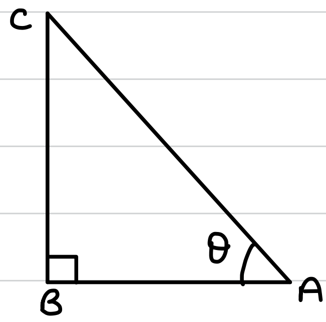
$\dfrac{CE}{DF} = k$

$\therefore \dfrac{AC}{BD} = \dfrac{AE}{BF} = \dfrac{CE}{DF} = k$

$\therefore \triangle ACE \sim \triangle BDF$ by SSS similarity

$\therefore \angle A = \angle B$ (corresponding parts of similar triangles)

7. $\cot\theta = \frac{7}{8}$



or $\frac{AB}{BC} = \frac{7}{8}$

Let $AB = 7k$, $BC = 8k$

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or $AC^2 = (7k)^2 + (8k)^2$

or $AC^2 = 49k^2 + 64k^2$

or $AC^2 = 113k^2$

or $AC = \sqrt{113k^2}$

or $AC = \sqrt{113}\,k$

$\sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}\,k} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}\,k} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$

$= \frac{(1)^2-\left(\frac{8}{\sqrt{113}}\right)^2}{(1)^2-\left(\frac{7}{\sqrt{113}}\right)^2}$

$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}}$

$= \frac{\frac{49}{113}}{\frac{64}{113}}$

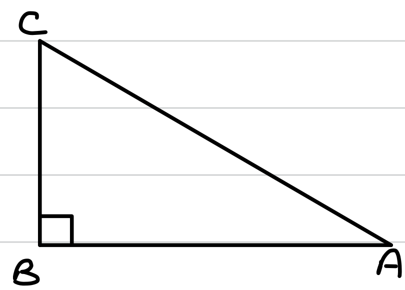
$= \frac{49}{\cancel{113}_1} \times \frac{\cancel{113}^1}{64}$

$= \frac{49}{64}$

(ii) $\cot^2\theta$

$= \left(\frac{7}{8}\right)^2$

$= \frac{49}{64}$



8\. $3 \cot A = 4$

or $\cot A = \frac{4}{3}$

or $\frac{AB}{BC} = \frac{4}{3}$

Let $AB = 4k$, $BC = 3k$

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or $AC^2 = (4k)^2 + (3k)^2$

or $AC^2 = 16k^2 + 9k^2$

or $AC^2 = 25k^2$

or $AC = \sqrt{25k^2}$

or $AC = 5k$

$\tan A = \frac{1}{\cot A} = \frac{3}{4}$

$\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

$L.H.S. = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$

$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$

$= \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$

$= \frac{\frac{7}{16}}{\frac{25}{16}}$

$= \frac{7}{\cancel{16}_1} \times \frac{\cancel{16}^1}{25}$

$= \frac{7}{25}$

$R.H.S. = \cos^2 A - \sin^2 A$

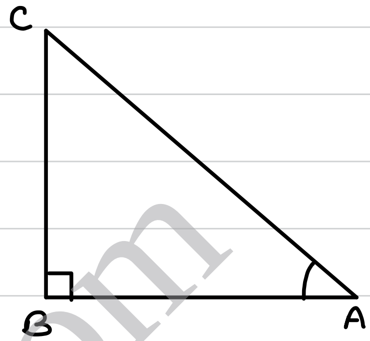
$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

$\therefore$ L.H.S. = R.H.S.

$\therefore \dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$



9. $\tan A = \dfrac{1}{\sqrt{3}}$

or $\dfrac{BC}{AB} = \dfrac{1}{\sqrt{3}}$

Let $BC = k$, $AB = \sqrt{3}\,k$

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or $AC^2 = (\sqrt{3}\,k)^2 + k^2$

or $AC^2 = 3k^2 + k^2$

or $AC^2 = 4k^2$

or $AC = 2k$

$\sin A = \dfrac{BC}{AC} = \dfrac{k}{2k} = \dfrac{1}{2}$ $\qquad$ $\sin C = \dfrac{AB}{AC} = \dfrac{\sqrt{3}\,k}{2k} = \dfrac{\sqrt{3}}{2}$

$\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{3}\,k}{2k} = \dfrac{\sqrt{3}}{2}$ $\qquad$ $\cos C = \dfrac{BC}{AC} = \dfrac{k}{2k} = \dfrac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

$= \dfrac{1}{2} \times \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{1}{4} + \dfrac{3}{4}$

$= \dfrac{\cancel{4}^{1}}{\cancel{4}_{1}}$

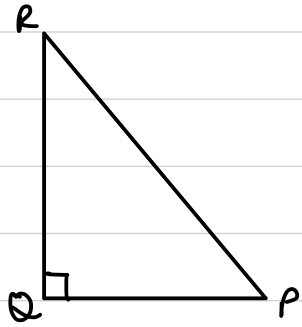
$= 1$

(ii) $\cos A \cos C - \sin A \sin C$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{1}{2} - \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{3}}{4} - \dfrac{\sqrt{3}}{4}$

$= 0$



10. Given - A ΔPQR, $\angle Q = 90°$, $PR + QR = 25\,cm$, $PQ = 5\,cm$

Solution - $PR + QR = 25\ cm$ (given)

or $PR = 25 - QR$ — ①

Using Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

or $(25 - QR)^2 = 5^2 + QR^2$ [Using equation ① and $PQ = 5\,cm$]

or $(25)^2 + QR^2 - 2 \times 25 \times QR = 25 + QR^2$

or $625 + QR^2 - 50\,QR = 25 + QR^2$

or $625 - 25 + \cancel{QR^2} - \cancel{QR^2} = 50\,QR$

or $600 = 50\,QR$

or $QR = \frac{\cancel{600}^{12}}{\cancel{50}_{1}}$

or $QR = 12\ cm$

Put the value of QR in equation ①

$PR = 25 - 12$

or $PR = 13\ cm$

$\sin P = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{QR}{PQ} = \frac{12}{5}$

11(i) False, the value of tan A varies from 0 to 1 as θ varies from 0° to 90°.

(ii) True, $\sec A = \frac{12}{5} > 1$

(iii) False, cosec A is the abbreviation for cosecant A.

(iv) False, cot A is not the product of cot and A. cot A is the relation between the angle A and sides of the triangle.

(v) False, $\sin\theta \neq \frac{4}{3}$ as $\frac{4}{3} > 1$. $0 \leq \sin\theta \leq 1$ for

$0^\circ \leq \theta \leq 90^\circ$