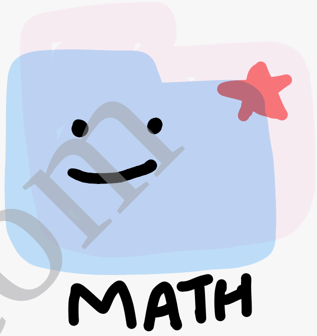
Coordinate Geometry
Ex. 7.1



Ex. 7.1

(i) Let the points be A(2,3) and B(4,1)

$AB = \sqrt{(4-2)^2+(1-3)^2}$

$= \sqrt{2^2+(-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

(ii) Let the points be A(−5,7) and B(−1,3)

$AB = \sqrt{[-1-(-5)]^2+(3-7)^2}$

$= \sqrt{4^2+(-4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

(iii) Let the points be P(a,b) and Q(−a,−b)

$PQ = \sqrt{(-a-a)^2+(-b-b)^2}$

$= \sqrt{(-2a)^2+(-2b)^2}$

$= \sqrt{4a^2+4b^2}$

$= \sqrt{4(a^2+b^2)}$

$= 2\sqrt{a^2+b^2}$

2. Let the points be A(0,0) and B(36,15)

$AB = \sqrt{(36-0)^2+(15-0)^2}$

$= \sqrt{36^2+15^2}$

$= \sqrt{1296+225}$

$= \sqrt{1521}$

$= 39$

The distance between towns A and B is 39 km.

3. Let the points be $A(1,5)$, $B(2,3)$ and $C(-2,-11)$

$$AB = \sqrt{(2-1)^2+(3-5)^2}$$
$$= \sqrt{1^2+(-2)^2}$$
$$= \sqrt{1+4}$$
$$= \sqrt{5}$$

$$BC = \sqrt{(-2-2)^2+(-11-3)^2}$$
$$= \sqrt{(-4)^2+(-14)^2}$$
$$= \sqrt{16+196}$$
$$= \sqrt{212}$$

$$AC = \sqrt{(-2-1)^2+(-11-5)^2}$$
$$= \sqrt{(-3)^2+(-16)^2}$$
$$= \sqrt{9+256}$$
$$= \sqrt{265}$$

Since $AB + BC \neq CA$

$\therefore$ Points A, B, C are non-collinear.

4. Let the points be $A(5,-2)$, $B(6,4)$ and $C(7,-2)$

$$AB = \sqrt{(6-5)^2+[4-(-2)]^2}$$
$$= \sqrt{1^2+6^2}$$
$$= \sqrt{1+36}$$
$$= \sqrt{37}$$

$$BC = \sqrt{(7-6)^2+(-2-4)^2}$$
$$= \sqrt{1^2+(-6)^2}$$
$$= \sqrt{1+36}$$
$$= \sqrt{37}$$

Since $AB = BC$

$\therefore (5,-2), (6,4), (7,-2)$ are the vertices of an isosceles triangle.

5. The coordinates of points are A(3, 4), B(6, 7), C(9, 4) and D(6, 1)

$AB = \sqrt{(6-3)^2+(7-4)^2} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2+(4-7)^2} = \sqrt{3^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2+(1-4)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$DA = \sqrt{(3-6)^2+(4-1)^2} = \sqrt{(-3)^2+3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(9-3)^2+(4-4)^2} = \sqrt{6^2+0^2} = \sqrt{36} = 6$

$BD = \sqrt{(6-6)^2+(1-7)^2} = \sqrt{0^2+(-6)^2} = \sqrt{36} = 6$

Since $AB = BC = CD = DA$

and diagonal AC = diagonal BD

$\therefore$ ABCD is a square.

$\therefore$ Champa is correct.

6(i) Let the points be A(−1, −2), B(1, 0), C(−1, 2), D(−3, 0)

$AB = \sqrt{[1-(-1)]^2+[0-(-2)]^2} = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2+(2-0)^2} = \sqrt{(-2)^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{[-3-(-1)]^2+(0-2)^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$DA = \sqrt{[-1-(-3)]^2+(-2-0)^2} = \sqrt{2^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AC = \sqrt{[-1-(-1)]^2+[2-(-2)]^2} = \sqrt{0^2+4^2} = \sqrt{16} = 4$

$BD = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{(-4)^2+0^2} = \sqrt{16} = 4$

Since $AB = BC = CD = DA$

and diagonal AC = diagonal BD

$\therefore$ ABCD is a square.

(ii) Let the points be A(−3, 5), B(3, 1), C(0, 3), D(−1, −4)

$AB = \sqrt{[3-(-3)]^2+(1-5)^2} = \sqrt{6^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(0-3)^2+(3-1)^2} = \sqrt{(-3)^2+2^2} = \sqrt{9+4} = \sqrt{13}$

$CD=\sqrt{(-1-0)^2+(-4-3)^2}=\sqrt{(-1)^2+(-7)^2}=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$

$DA=\sqrt{[-3-(-1)]^2+[5-(-4)]^2}=\sqrt{(-2)^2+9^2}=\sqrt{4+81}=\sqrt{85}$

Since $AB \neq BC \neq CD \neq DA$

$\therefore$ ABCD is a quadrilateral.

(iii) Let the points be $A(4,5)$, $B(7,6)$, $C(4,3)$, $D(1,2)$

$AB=\sqrt{(7-4)^2+(6-5)^2}=\sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}$

$BC=\sqrt{(4-7)^2+(3-6)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

$CD=\sqrt{(1-4)^2+(2-3)^2}=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$

$DA=\sqrt{(4-1)^2+(5-2)^2}=\sqrt{3^2+3^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

$AC=\sqrt{(4-4)^2+(3-5)^2}=\sqrt{0^2+(-2)^2}=\sqrt{4}=2$

$BD=\sqrt{(1-7)^2+(2-6)^2}=\sqrt{(-6)^2+(-4)^2}=\sqrt{36+16}=\sqrt{52}=2\sqrt{13}$

Since $AB=CD$ and $BC=AD$ and

diagonal $AC \neq$ diagonal BD

$\therefore$ ABCD is a parallelogram.

7. Let $P(x,0)$ be equidistant from $A(2,-5)$ and $B(-2,9)$.

$\therefore AP = BP$

or $\sqrt{(x-2)^2+[0-(-5)]^2}=\sqrt{[x-(-2)]^2+(0-9)^2}$

Squaring both sides

$(x-2)^2+5^2=(x+2)^2+(-9)^2$

or $x^2+4-4x+25=x^2+4+4x+81$

or $x^2-4x+29=x^2+4x+85$

or $\cancel{x^2}-4x-\cancel{x^2}-4x=85-29$

or $-8x=56$

or $x=\frac{\cancel{56}\,7}{\cancel{-8}\,1}$

or $x=-7$

$\therefore$ P(−7, 0) is equidistant from A(2, −5) and B(−2, 9).

8. $P(2,-3),\ Q(10,y)$

$PQ = 10$ (given)

or $\sqrt{(10-2)^2+[y-(-3)]^2} = 10$

Squaring both sides

$8^2+(y+3)^2 = 10^2$

or $64+y^2+9+6y = 100$

or $y^2+6y+73-100=0$

or $y^2+6y-27=0$

or $y^2+9y-3y-27=0$

or $y(y+9)-3(y+9)=0$

or $(y+9)(y-3)=0$

either $y-3=0$ or $y+9=0$

or $y=3$ or $y=-9$

$\therefore\ y = 3$ or -9

9. $P(5,-3),\ Q(0,1),\ R(x,6)$

$PQ = QR$ (given)

or $\sqrt{(0-5)^2+[1-(-3)]^2} = \sqrt{(x-0)^2+(6-1)^2}$

Squaring both sides

$(-5)^2+4^2 = x^2+5^2$

or $25+16 = x^2+25$

or $41-25=x^2$

or $16=x^2$

Taking square root on both sides

$x = \pm 4$

When $x=4$, $R(4,6)$

$P(5,-3) \quad Q(0,1) \quad R(-4,6)$

$QR = \sqrt{(4-0)^2+(6-1)^2} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(4-5)^2+[6-(-3)]^2} = \sqrt{(-1)^2+9^2} = \sqrt{1+81} = \sqrt{82}$

When $x=-4$, $R(-4,6)$

$PR = \sqrt{[5-(-4)]^2+(-3-6)^2} = \sqrt{9^2+(-9)^2} = \sqrt{81+81} = 9\sqrt{2}$

$QR = \sqrt{(-4-0)^2+(6-1)^2} = \sqrt{(-4)^2+5^2} = \sqrt{16+25} = \sqrt{41}$

10. Let $P(x,y)$ be equidistant from $A(3,6)$ and $B(-3,4)$

$AP = BP$ (given)

or $\sqrt{(x-3)^2+(y-6)^2} = \sqrt{[x-(-3)]^2+(y-4)^2}$

Squaring both sides

$x^2+9-6x+y^2+36-12y = x^2+9+6x+y^2+16-8y$

or $x^2+y^2-6x-12y+45 = x^2+y^2+6x-8y+25$

or $\cancel{x^2}+\cancel{y^2}-6x-12y+45-\cancel{x^2}-\cancel{y^2}-6x+8y-25=0$

or $-12x-4y+20=0$

Dividing both sides by -4

$3x+y-5=0$