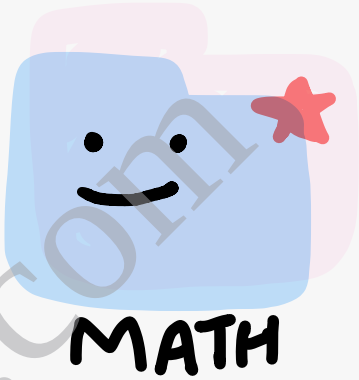
Squares, Cubes And Their Roots

Ex. 3.1



Ex. 3.1

1.a $2061 = 3 \times 3 \times 229$

Since 229 does not occur in a pair.

$\therefore$ 2061 is not a perfect square.

$3 \mid 2061$

$3 \mid 687$

229

b.

$2 \mid 7928$

$2 \mid 3964$

$2 \mid 1982$

991

$7928 = 2 \times 2 \times 2 \times 991$

Since 2 and 991 do not occur in pairs.

$\therefore$ 7928 is not a perfect square.

c.

$7 \mid 2401$

$7 \mid 343$

$7 \mid 49$

7

$2401 = 7 \times 7 \times 7 \times 7$

Since 7 occurs in a pair.

$\therefore$ 2401 is a perfect square.

d.

$5 \mid 445$

89

$445 = 5 \times 89$

Since 5 and 89 do not occur in a pair.

$\therefore$ 445 is not a perfect square

2.a $16^2 = 256$

b. $(-32)^2 = 1024$

c. $35^2 = 1225$

d. $(-46)^2 = 2116$

e. $50^2 = 2500$

f. $65^2 = 4225$

g. $(-75)^2 = 5625$

h. $89^2 = 7921$

3.
$$\begin{array}{r|l} 7 & 539 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

$539 = \underline{7 \times 7} \times 11$

Since 11 does occur in a pair.

$\therefore$ 539 should be multiplied by 11 so that the product is a perfect square.

$\therefore$ Smallest number = 11

4.
$$\begin{array}{r|l} 2 & 13068 \\ \hline 2 & 6534 \\ \hline 3 & 3267 \\ \hline 3 & 1089 \\ \hline 3 & 363 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

$13068 = \underline{2 \times 2} \times \underline{3 \times 3} \times 3 \times \underline{11 \times 11}$

3 does not occur in a pair

$\therefore$ 13068 should be divided by 3 so that the quotient is a perfect square.

$\therefore$ Smallest number = 3

5.a. 11^2 will have 1 at its unit's place as the digit at unit's place is 1 and $(1)^2 = 1$.

b. 35^2 will not have 1 at its unit's place as the digit at unit's place is 5 and $(5)^2 = 25$.

c. 49^2 will have 1 at its unit's place as the digit at unit's place is 9 and $(9)^2 = 81$.

d. 51^2 will have 1 at its unit's place as the digit at unit's place is 1 and $(1)^2 = 1$.

e. 66^2 will not have 1 at its unit's place as the digit at unit's place is 6 and $(6)^2 = 36$.

f. 79^2 will have 1 at its unit's place as the digit at unit's place is 9 and $(9)^2 = 81$.

g. 81^2 will have 1 at its unit's place as the digit at unit's place is 1 and $(1)^2 = 1$.

h. 95^2 will not have 1 at its unit's place as the digit at unit's place is 5 and $(5)^2 = 25$.

6.a. $36 = 1+3+5+7+9+11$

b. $64 = 1+3+5+7+9+11+13+15$

c. $81 = 1+3+5+7+9+11+13+15+17$

d. $144 = 1+3+5+7+9+11+13+15+17+19+21+23$

7.a. $2n = 14$

or $n = \frac{14}{2}$ 7

or $n = 7$

$n^2 - 1 = 7^2 - 1 = 49 - 1 = 48$

$n^2 + 1 = 7^2 + 1 = 49 + 1 = 50$

$\therefore$ Pythagorean triplet is 14, 48, 50.

b. $2n = 10$

or $n = \frac{10}{2}^{5}_{1}$

or $n = 5$

$n^2 - 1 = 5^2 - 1 = 25 - 1 = 24$

$n^2 + 1 = 5^2 + 1 = 25 + 1 = 26$

$\therefore$ Pythagorean triplet is 10, 24, 26.

c. $2n = 16$

or $n = \frac{16}{2}^{8}_{1}$

or $n = 8$

$n^2 - 1 = 8^2 - 1 = 64 - 1 = 63$

$n^2 + 1 = 8^2 + 1 = 64 + 1 = 65$

$\therefore$ Pythagorean triplet is 16, 63, 65.

d. $2n = 22$

or $n = \frac{22}{2}^{11}_{1}$

or $n = 11$

$n^2 - 1 = 11^2 - 1 = 121 - 1 = 120$

$n^2 + 1 = 11^2 + 1 = 121 + 1 = 122$

$\therefore$ Pythagorean triplet is 22, 120, 122.

8a. $(15)^2$

Here $n = 15$

$\frac{n^2 - 1}{2} = \frac{15^2 - 1}{2} = \frac{225 - 1}{2} = \frac{224}{2} = 112$

$\frac{n^2 + 1}{2} = \frac{15^2 + 1}{2} = \frac{225 + 1}{2} = \frac{226}{2} = 113$

$\therefore 15^2 = 225 = 112 + 113$

b. $11^2 = 121$

Let the two consecutive numbers be x and $x+1$

$\therefore x + x + 1 = 121$

or $2x = 121 - 1$

or $2x = 120$

or $x = \frac{\cancel{120}}{\cancel{2}_1}^{60}$

or $x = 60$

$\therefore 11^2 = 121 = 60 + 61$

c. $(19)^2 = 361 = 180 + 181$

d. $(21)^2 = 441 = 220 + 221$

9a. $21^2 - 20^2 = 21 + 20 = 41$ as $(n+1)^2 - n^2 = (n+1) + n = 2n + 1$

b. $34^2 - 33^2 = 34 + 33 = 67$

c. $95^2 - 94^2 = 95 + 94 = 189$

10a. The digit at unit's place of 46^2 is 6 as $6^2 = 36$.

b. The digit at unit's place of 50^2 is 0.

c. The digit at unit's place of 97^2 is 9 as $7^2 = 49$.

d. The digit at unit's place of 132^2 is 4 as $2^2 = 4$.

e. The digit at unit's place of 258^2 is 4 as $8^2 = 64$.

f. The digit at unit's place of 315^2 is 5 as $5^2 = 25$.

g. The digit at unit's place of 442^2 is 4 as $2^2 = 4$.

h. The digit at unit's place of 525^2 is 5 as $5^2 = 25$.

11. 22, 243, 347, 548 are not perfect squares as a number with 2, 3, 7, 8 at unit's place is not a perfect square.

12a. $1+3+5+7+9+11 = 6^2 = 36$

b. $1+3+5+7+9+11+13+15+17+19 = 10^2 = 100$

c. $1+3+5+7+9+11+13+15+17+19+21+23+25+27 = 15^2 = 225$

d. $1+3+5+7+9+11+13+15+17+19+21+23+25+27+29+31 = 17^2 = 289$

13a. 9 is square of an odd number.

b. 36 is square of an even number.

c. 100 is square of an even number.

d. 169 is square of an even number.

e. 256 is square of an odd number.

f. 361 is square of an even number.

g. 484 is square of an even number.

h. 576 is square of an even number.

(The square of an even number is even and the square of an odd number is odd)