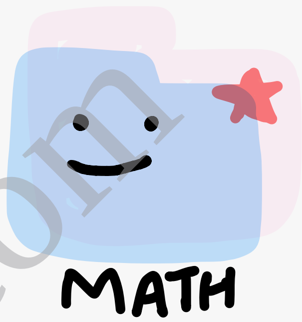
Arithmetic Progressions
Ex. 5.4



$+ \quad \times$

$- \quad \div$

Ex. 5.4

1. $121, 117, 113, ----$

$a = 121,\ d = 117 - 121 = -4$

Let the n^{th} term be the first negative term.

$\therefore a_n < 0$

or $a + (n-1)d < 0$

or $121 + (n-1)(-4) < 0$

or $(n-1)(-4) < -121$

or $n - 1 > \frac{121}{4}$

or $n > \frac{121}{4} + 1$

or $n > \frac{125}{4}$

or $n > 31.25$

$\therefore$ The first negative is 32^{nd} term.

2. Let the first term be 'a' and 'd' be the common difference.

$a_3 + a_7 = 6$ (given)

or $a + 2d + a + 6d = 6$

or $2a + 8d = 6$

Dividing both sides by 2

$a + 4d = 3$

or $a = 3 - 4d$ — ①

$a_3 \times a_7 = 8$ (given)

or $(a + 2d)(a + 6d) = 8$

or $(3 - 4d + 2d)(3 - 4d + 6d) = 8$ (using equation ①)

or $(3 - 2d)(3 + 2d) = 8$

or $(3)^2 - (2d)^2 = 8$ $[\because (a-d)(a+d) = a^2 - d^2]$

or $9-4d^2=8$

or $9-8=4d^2$

or $1=4d^2$

or $d^2=\frac{1}{4}$

Taking square root on both sides

$d=\pm\frac{1}{2}$

When $d=\frac{1}{2}$

From eq. 1

$a=3-\cancel{4}^{2}\times\frac{1}{\cancel{2}}$

or $a=1$

$S_{16}=\frac{\cancel{16}^{8}}{\cancel{2}_{1}}\left[2\times1+15\times\frac{1}{2}\right]$

$=8\left(2+\frac{15}{2}\right)$

$=\cancel{8}^{4}\times\frac{19}{\cancel{2}_{1}}$

$=76$

When $d=-\frac{1}{2}$

From eq. 1

$a=3-\cancel{4}^{2}\left(-\frac{1}{\cancel{2}_{1}}\right)$

$a=5$

$S_{16}=\frac{\cancel{16}^{8}}{\cancel{2}_{1}}\left[2\times5+15\left(-\frac{1}{2}\right)\right]$

$=8\left(10-\frac{15}{2}\right)$

$=\cancel{8}^{4}\left(\frac{5}{\cancel{2}_{1}}\right)$

$=20$

$\therefore$ Sum of 16 terms $=76$ or 20.

3. Distance between consecutive rungs $=25$ cm

Length of longest rung $=45$ cm

Length of shortest rung $=25$ cm

Distance between top and bottom rung $=2\frac{1}{2}$ m

$=\frac{5}{\cancel{2}_{1}}\times\cancel{100}^{50}$ cm $=250$ cm

Number of rungs $= \frac{\cancel{250}^{10}}{\cancel{25}_{1}} + 1$

$= 11$

The length of rungs forms an A.P. with first term = 45 and last term = 25 and d = 25 cm

$\therefore$ length of wood required $= S_{11}$

$= \frac{11}{2}(45+25)$

$= \frac{11}{\cancel{2}_{1}} \times \cancel{70}^{35}$

$= 385$ cm

4. The house numbers 1, 2, 3, 4, 5, – – –, 49 form an A.P. with $a = 1$, $d = 1$

Sum of numbers of houses preceding house number $x = S_{x-1}$

Sum of numbers of houses after house number x
$= S_{49} - S_x$

$\therefore$ According to the given condition

$S_{x-1} = S_{49} - S_x$

or $\left(\frac{x-1}{2}\right)(\cancel{1} + x - \cancel{1}) = \frac{49}{2}(1+49) - \frac{x}{2}(1+x)$

Multiplying both sides by 2

$(x-1)x = 49 \times 50 - x(1+x)$

or $x^2 - x = 2450 - x^2 - x$

or $x^2 - \cancel{x} + x^2 + \cancel{x} = 2450$

or $2x^2 = 2450$

or $x^2 = 1225$

Taking square root on both sides

$x = \pm 35$

We reject (-35) as house number cannot be negative.

$\therefore x = 35$

5. Total number of steps $= 15$

Length of each step, $l = 50\,m$

Width of each step, $b = \frac{1}{2}\,m$

Height of each step, $h = \frac{1}{4}\,m$

Volume of concrete required to build the first step $= l\,b\,h$

$= \overset{25}{\cancel{50}} \times \frac{1}{\cancel{2}} \times \frac{1}{4}$

$= \frac{25}{4}\,m^3$

Height of second step $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\,m$

Volume of concrete required to build the second step $= \overset{25}{\cancel{50}} \times \frac{1}{\underset{1}{\cancel{2}}} \times \frac{1}{2} = \frac{25}{2}\,m^3$

Height of third step $= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}\,m$

Volume of concrete required to build the third step $= \overset{25}{\cancel{50}} \times \frac{1}{\cancel{2}} \times \frac{3}{4} = \frac{75}{4}\,m^3$

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$a_2 - a_1 = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$

$$a_3 - a_2 = \frac{75}{4} - \frac{25}{2} = \frac{25}{2}\left(\frac{3}{2} - 1\right) = \frac{25}{2} \times \frac{1}{2} = \frac{25}{4}$$

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Since $a_{k+1} - a_k$ is same for all the values of k

$\therefore \frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$ form an A.P. with $a = \frac{25}{4}$

and $d = \frac{25}{4}$

$\therefore$ Volume of concrete required for 15 steps $= S_{15}$

$$= \frac{15}{2}\left[\cancel{2} \times \frac{25}{\cancel{4}_{2}} + \overset{7}{\cancel{14}} \times \frac{25}{\cancel{4}_{2}}\right]$$

$$= \frac{15}{2}\left(\frac{25}{2} + \frac{175}{2}\right)$$

$$= \frac{15}{\cancel{2}} \times \frac{\cancel{200}\,\cancel{100}\,50}{\cancel{2}}$$

$$= 750 \text{ m}^3$$