Arithmetic Progressions Ex. 5.3

$+ \quad \times$

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Ex. 5.3

1 (i) 2, 7, 12, ---- to 10 terms

$a = 2,\ d = 7 - 2 = 5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{\cancel{10}^{5}}{\cancel{2}}[2 \times 2 + (10-1)5]$

$= 5\ (4 + 9 \times 5)$

$= 5 \times 49$

$= 245$

(ii) $-37, -33, -29,$ ---- to 12 terms

$a = -37,\ d = -33 - (-37) = -33 + 37 = 4$

$S_{12} = \frac{\cancel{12}^{6}}{\cancel{2}}[2 \times (-37) + (12-1)4]$

$= 6\ (-74 + 44)$

$= 6\ (-30)$

$= -180$

(iii) 0.6, 1.7, 2.8, ---- to 100 terms

$a = 0.6,\ d = 1.7 - 0.6 = 1.1$

$S_{100} = \frac{\cancel{100}^{50}}{\cancel{2}}[2 \times 0.6 + (100-1)1.1]$

$= 50\ (1.2 + 108.9)$

$= 50 \times 110.1$

$= 5505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10},$ ---- to 11 terms

$a = \frac{1}{15},\ d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

$$S_{11} = \frac{11}{2}\left[2 \times \frac{1}{15} + (11-1) \times \frac{1}{60}\right]$$

$$= \frac{11}{2}\left(\frac{2}{15} + \frac{10}{60}\right)$$

$$= \frac{11}{2}\left(\frac{8+10}{60}\right)$$

$$= \frac{11}{\cancel{2}} \times \frac{\cancel{18}^{\cancel{9}\,3}}{\cancel{60}_{20}}$$

$$= \frac{33}{20}$$

20 (i) $7 + 10\frac{1}{2} + 14 + \text{-----} + 84$

$a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2}$

$a_3 - a_2 = 14 - 10\frac{1}{2} = \frac{28-21}{2} = \frac{7}{2}$

— — — — — — —

Since $a_{k+1} - a_k$ is same for all the values of k

$\therefore$ $7, 10\frac{1}{2}, 14, \text{---}$ form an A.P with $a = 7$, $d = \frac{7}{2}$

Let the number of terms be n.

$\therefore$ $a_n = 84$

or $a + (n-1)d = 84$

or $7 + (n-1)\frac{7}{2} = 84$

or $n - 1 = \frac{\cancel{77}^{11} \times 2}{\cancel{7}}$

or $n = 23$

$\therefore S_{23} = \frac{23}{2}\left[2\times 7 + (23-1)\frac{7}{2}\right]$ <u>OR</u> $S_{23} = \frac{23}{2}(7+84)$

$= \frac{23}{2}\left(14 + \cancel{22}^{11}\times\frac{7}{\cancel{2}}\right)$ $= \frac{23}{2}\times 91$

$= \frac{23}{2}\times 91$ $= \frac{2093}{2}$

$= \frac{2093}{2}$ $= 1046\frac{1}{2}$

$= 1046\frac{1}{2}$

(11) $34+32+30+ ---- +10$

$a_2 - a_1 = 32-34 = -2$

$a_3 - a_2 = 30-32 = -2$

$- - - - - - - - -$

$- - - - - - - -$

Since $a_{k+1} - a_k$ is same for all the values of k

$\therefore$ 34, 32, 30, ---- form an A.P. with $a = 34$, $d = -2$

Let the number of terms be n.

$\therefore a_n = 10$

or $a + (n-1)d = 10$

or $34 + (n-1)(-2) = 10$

or $n - 1 = \frac{-\cancel{24}^{12}}{-\cancel{2}}$

or $n = 13$

$\therefore S_{13} = \frac{13}{2}(34+10)$

$= \frac{13}{\cancel{2}}\times \cancel{44}^{22}$

$= 286$

(iii) $-5+(-8)+(-11)+ ---- +(-230)$

$a_2-a_1 = -8-(-5) = -8+5 = -3$

$a_3-a_2 = -11-(-8) = -11+8 = -3$

Since $a_{k+1}-a_k$ is same for all the values of k.

$\therefore$ $-5, -8, -11, ----$ form an A.P. with $a=-5, d=-3$

Let the number of terms be n.

$\therefore$ $a_n = -230$

or $a+(n-1)d = -230$

or $-5+(n-1)(-3) = -230$

or $n-1 = \frac{\cancel{-225}\ 75}{\cancel{-3}}$

or $n = 76$

$\therefore$ $S_{76} = \frac{\overset{38}{\cancel{76}}}{\cancel{2}}[-5+(-230)]$

$= 38(-235)$

$= -8930$

30(i) $a=5,\ d=3,\ a_n=50,\ n=?,\ S_n=?$

$a_n = 50$

or $a+(n-1)d = 50$

or $5+(n-1)3 = 50$

or $n-1 = \frac{\cancel{45}\ 15}{\cancel{3}}$

or $n = 16$

$S_n = \frac{n}{2}(a+a_n)$

$= \frac{\overset{8}{\cancel{16}}}{\cancel{2}}(5+50)$

$= 8 \times 55$

$= 440$

(ii) $a=7,\ a_{13}=35,\ d=?\ ,\ S_{13}=?$

$a_{13}=35$

or $a+12d=35$

or $7+12d=35$

or $12d=28$

or $d=\frac{\cancel{28}\,7}{\cancel{12}\,3}$

or $d=\frac{7}{3}$

$S_{13}=\frac{13}{2}(7+35)$

$=\frac{13}{\cancel{2}}\times\cancel{42}^{21}$

$=273$

(iii) $a_{12}=37,\ d=3,\ a=?\ ,\ S_{12}=?$

$a_{12}=37$

or $a+11d=37$

or $a+11\times3=37$

or $a=37-33$

or $a=4$

$S_{12}=\frac{\cancel{12}^{6}}{\cancel{2}}(4+37)$

$=6\times41$

$=246$

(iv) $a_3=15,\ S_{10}=125,\ d=?\ ,\ a_{10}=?$

$a_3=15$

$a+2d=15 \Rightarrow a=15-2d$ — (1)

$S_{10} = 125$

or $\frac{\cancel{10}^{5}}{\cancel{2}}(2a+9d) = 125$

or $2a+9d = \frac{\cancel{125}^{25}}{\cancel{5}}$

or $2a+9d = 25$ — (ii)

Put the value of a from eq. (i) in eq. (ii)

$2(15-2d) + 9d = 25$

or $30 - 4d + 9d = 25$

or $5d = -5$

or $d = -1$

Put the value of d in eq. (i)

$a = 15 - 2(-1)$

or $a = 17$

$\therefore a_{10} = a + 9d$

$= 17 + 9(-1)$

$= 17 - 9$

$= 8$

(v) $d = 5, S_9 = 75, a = ?, a_9 = ?$

$S_9 = 75$

or $\frac{9}{2}(2a + 8\times5) = 75$

or $\frac{9}{\cancel{2}_{1}} \times \cancel{2}^{1}(a+20) = 75$

Dividing both sides by 3

$3(a+20) = 25$

or $3a + 60 = 25$

or $3a = 25 - 60$

or $a = -\frac{35}{3}$

$a_9 = a + 8d$

$= -\frac{35}{3} + 8 \times 5$

$= \frac{-35+120}{3}$

$= \frac{85}{3}$

(vi) $a = 2,\ d = 8,\ S_n = 90,\ n = ?,\ a_n = ?$

$S_n = 90$

or $\frac{n}{2}[2a + (n-1)d] = 90$

or $\frac{n}{2}[2 \times 2 + (n-1)8] = 90$

or $\frac{n}{\cancel{2}_1} \times \cancel{2}^1 [2 + (n-1)4] = 90$

or $n(2 + 4n - 4) = 90$

or $n(4n - 2) = 90$

or $4n^2 - 2n - 90 = 0$

Dividing both sides by 2

$2n^2 - n - 45 = 0$

or $2n^2 - 10n + 9n - 45 = 0$

or $2n(n-5) + 9(n-5) = 0$

or $(n-5)(2n+9) = 0$

either $n - 5 = 0$ or $2n + 9 = 0$

or $n = 5$ or $n = -\frac{9}{2}$ We reject $\left(-\frac{9}{2}\right)$ as n is not a positive integer

$\therefore n = 5$

$\therefore a_5 = a + 4d$

$= 2 + 4 \times 8$

$= 34$

(vii) $a = 8$, $a_n = 62$, $S_n = 210$, $n = ?$, $d = ?$

$S_n = 210$

$\frac{n}{2}(a + a_n) = 210$

or $\frac{n}{2}(8 + 62) = 210$

or $\frac{n}{\cancel{2}} \times \overset{35}{\cancel{70}} = 210$

or $n = \frac{\overset{6}{\cancel{210}}}{\underset{1}{\cancel{35}}}$

or $n = 6$

$\therefore a_6 = 62$ (given)

or $a + 5d = 62$

or $8 + 5d = 62$

or $5d = 54$

or $d = \frac{54}{5}$

(viii) $a_n = 4$, $d = 2$, $S_n = -14$, $n = ?$, $a = ?$

$a_n = 4$

or $a + (n-1)d = 4$

or $a + (n-1)2 = 4$

or $a + 2n - 2 = 4$

or $a + 2n = 6$

or $a = 6 - 2n$ — (1)

$S_n = -14$

or $\frac{n}{2}[2a+(n-1)d] = -14$

or $\frac{n}{2}[2a+(n-1)2] = -14$

or $\frac{n}{\cancel{2}_1} \times \cancel{2}^1 [a+n-1] = -14$

or $n\ (6-2n+n-1) = -14$ (using eq. ①)

or $n\ (5-n) = -14$

or $5n - n^2 = -14$

or $0 = n^2 - 5n - 14$

or $n^2 - 7n + 2n - 14 = 0$

or $n(n-7) + 2(n-7) = 0$

or $(n-7)(n+2) = 0$

either $n-7=0$ or $n+2=0$

or $n=7$ or $n=-2$ We reject (-2) as it is not a positive integer

$\therefore n = 7$

$a_n = 4$ (given)

or $a_7 = 4$

or $a+6d = 4$

or $a + 6\times 2 = 4$

or $a = 4-12$

or $a = -8$

(ix) $a=3,\ n=8,\ S=192,\ d=?$

$S_8 = 192$ (given)

or $\frac{\cancel{8}^4}{\cancel{2}_1}[2\times 3 + 7d] = 192$

or $6 + 7d = \frac{\cancel{192}^{48}}{\cancel{4}_1}$

or $7d = 42$

or $d = \frac{\cancel{42}^{6}}{\cancel{7}_1}$

or $d = 6$

(x) $l = 28$, $S = 144$, $n = 9$, $a = ?$

$S_9 = 144$

or $\frac{9}{2}(a + 28) = 144$

or $a + 28 = \frac{\cancel{144}^{16} \times 2}{\cancel{9}_1}$

or $a = 32 - 28$

or $a = 4$

4. $9, 17, 25, ----$

$a = 9$, $d = 17 - 9 = 8$

Let the sum of n terms be 636.

$\therefore S_n = 636$

or $\frac{n}{2}[2 \times 9 + (n-1)8] = 636$

or $\frac{n}{\cancel{2}_1} \times \cancel{2}^1 [9 + (n-1)4] = 636$

or $n(9 + 4n - 4) = 636$

or $n(5 + 4n) = 636$

or $5n + 4n^2 = 636$

or $4n^2 + 5n - 636 = 0$

or $4n^2 + 53n - 48n - 636 = 0$

or $n(4n+53)-12(4n+53)=0$

or $(4n+53)(n-12)=0$

either $4n+53=0$ or $n-12=0$

or $n=-\frac{53}{4}$ or $n=12$

We reject $\left(-\frac{53}{4}\right)$ as n is not a positive integer.

$\therefore n=12$

$\therefore$ Sum of 12 terms is 636.

5. $a=5$

Let the number of terms be n.

$\therefore a_n=45$

$S_n=400$

or $\frac{n}{2}(a+a_n)=400$

or $\frac{n}{2}(5+45)=400$

or $\frac{n}{\cancel{2}_1}\times\cancel{50}^{25}=400$

or $n=\frac{\cancel{400}^{16}}{\cancel{25}_1}$

or $n=16$

$a_n=45$ (given)

or $a_{16}=45$

or $a+15d=45$

or $5+15d=45$

or $15d=40$

or $d=\frac{\cancel{40}^{8}}{\cancel{15}_{3}}$ or $d=\frac{8}{3}$

6\. Let the number of terms be n.

$a = 17$

$d = 9$

$a_n = 350$

$a + (n-1)d = 350$

or $17 + (n-1)9 = 350$

or $(n-1)9 = 333$

or $n - 1 = \cancel{\frac{333}{9}}\ 37$

or $n = 38$

$S_{38} = \frac{\cancel{38}^{19}}{\cancel{2}_1}(17 + 350)$

or $S_{38} = 19 \times 367$

or $S_{38} = 6973$

7\. $d = 7$

$a_{22} = 149$

or $a + 21d = 149$

or $a + 21 \times 7 = 149$

or $a + 147 = 149$

or $a = 2$

$S_{22} = \frac{\cancel{22}^{11}}{\cancel{2}_1}(2 + 149)$

$= 11 \times 151$

$= 1661$

8\. Let 'a' be the first term and 'd' be the common difference.

$a_2 = 14 \Rightarrow a + d = 14$ — (1)

$a_3 = 18 \Rightarrow a + 2d = 18$ — (ii)

Subtracting eq. (i) from eq. (ii)

$\cancel{a} + 2d - \cancel{a} - d = 18 - 14$

or $d = 4$

Put the value of d in eq. (i)

$a + 4 = 14$

or $a = 10$

$S_{51} = \frac{51}{2}[2 \times 10 + 50 \times 4]$

$= \frac{51}{\cancel{2}_1} \times \overset{1}{\cancel{2}}[10 + 100]$

$= 51 \times 110$

$= 5610$

9. Let 'a' be the first term and 'd' be the common difference.

$S_7 = 49$

or $\frac{7}{2}(2a + 6d) = 49$

or $\frac{7}{\cancel{2}_1} \times \overset{1}{\cancel{2}}(a + 3d) = 49$

or $a + 3d = 7$ — (i)

$S_{17} = 289$

or $\frac{17}{2}(2a + 16d) = 289$

or $\frac{17}{\cancel{2}_1} \times \overset{1}{\cancel{2}}(a + 8d) = 289$

or $a + 8d = 17$ — (ii)

Subtracting eq. (i) from eq. (ii)

$a + 8d - (a + 3d) = 17 - 7$

or $\cancel{a} + 8d - \cancel{a} - 3d = 10$

or $5d = 10$

or $d = 2$

Put the value of d in eq. (i)

$a + 3 \times 2 = 7$

or $a = 7 - 6$

or $a = 1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{\cancel{2}} \times \cancel{2}(\cancel{1} + n - \cancel{1})$

$= n \times n$

$= n^2$

10 (i) $a_n = 3 + 4n$

$a_1 = 3 + 4 \times 1 = 7$

$a_2 = 3 + 4 \times 2 = 11$

$a_3 = 3 + 4 \times 3 = 15$

$a_4 = 3 + 4 \times 4 = 19$

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 7, 11, 15, 19, ---- form an A.P. with $a = 7, d = 4$

$S_{15} = \frac{15}{2}(2 \times 7 + 14 \times 4)$

$= \frac{15}{\cancel{2}} \times \overset{7}{\cancel{14}}(1 + 4)$

$= 105 \times 5$

$= 525$

(11) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

— — — — — — — — — —

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -6 + 1 = -5$

$a_4 - a_3 = -11 - (-6) = -11 + 6 = -5$

— — — — — — — — — —

Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 4, −1, −6, −11 form an A.P. with $a = 4$ and $d = -5$

$S_{15} = \frac{15}{2}\left[2 \times 4 + 14(-5)\right]$

$= \frac{15}{\cancel{2}_1} \times \cancel{2}^1 (4 - 35)$

$= 15(-31)$

$= -465$

11. $S_n = 4n - n^2$

$S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$

$S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$

$a_2 = S_2 - S_1$

$= 4 - 3$

$= 1$

$S_3 = 4 \times 3 - 3^2$

$= 12 - 9$

$= 3$

$a_3 = S_3 - S_2$

or $a_3 = 3-4$

or $a_3 = -1$

$S_{10} = 4\times10-(10)^2$

$= 40-100$

$= -60$

$S_9 = 4\times9-9^2$

$= 36-81$

$= -45$

$a_{10} = S_{10}-S_9$

$= -60-(-45)$

$= -60+45$

$= -15$

$S_n = 4n-n^2$

$S_{n-1} = 4(n-1)-(n-1)^2$

$= 4n-4-(n^2-2n+1)$ $[\because (a-b)^2 = a^2-2ab+b^2]$

$= 4n-4-n^2+2n-1$

$= -n^2+6n-5$

$a_n = S_n - S_{n-1}$

$= 4n-n^2-(-n^2+6n-5)$

$= 4n-\cancel{n^2}-\cancel{n^2}-6n+5$

$= 5-2n$

12. First 40 positive integers divisible by 6 are

6, 12, 18, 24, ------, 240

$a_2-a_1 = 12-6 = 6$

$a_3-a_2 = 18-12 = 6$

$a_4-a_3 = 24-18 = 6$

Since $a_{k+1}-a_k$ is same for all the values of k

$\therefore$ 6, 12, 18, 24, ----, 240 form an A.P. with $a = 6$ and $d = 6$

Sum of first 40 positive integers divisible by 6 $= S_{40}$

$$= \frac{\cancel{40}^{20}}{\cancel{2}_1}(6 + 240)$$

$$= 20 \times 246$$

$$= 4920$$

13. First 15 multiples of 8 are 8, 16, 24, 32, ----, 120

$a_2 - a_1 = 16 - 8 = 8$

$a_3 - a_2 = 24 - 16 = 8$

$a_4 - a_3 = 32 - 24 = 8$

-- -- -- --

Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 8, 16, 24, 32, ----, 120 form an A.P. with $a = 8$ and $d = 8$.

$\therefore$ Sum of first 15 multiples of 8 $= S_{15}$

$$= \frac{15}{2}(8 + 120)$$

$$= \frac{15}{\cancel{2}_1} \times \cancel{128}^{64}$$

$$= 960$$

14. Odd numbers between 0 and 50 are 1, 3, 5, 7, 9, ---, 49

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 5 - 3 = 2$

$a_4 - a_3 = 7 - 5 = 2$

-- -- -- --

Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 1, 3, 5, 7, 9, ---, 49 form an A.P. with $a = 1$ and $d = 2$.

Let the number of terms be n.

$\therefore a_n = 49$

or $a+(n-1)d = 49$

or $1+(n-1)2 = 49$

or $(n-1)2 = 48$

or $n-1 = \cancel{\frac{48}{2}}^{24}_{1}$

or $n = 24+1$

or $n = 25$

Sum of odd numbers between 0 and 50 $= S_{25}$

$= \frac{25}{2}(1+49)$

$= \frac{25}{\cancel{2}_1} \times \cancel{50}^{25}$

$= 625$

15. Penalty for first day = ₹ 200

Penalty for second day = ₹ 250

Penalty for third day = ₹ 300

— — — — — — — — — — —

$a_2 - a_1 = 250 - 200 = 50$

$a_3 - a_2 = 300 - 250 = 50$

— — — — — — — — — — —

Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 200, 250, 300, — — — — form an A.P. with $a = 200, d = 50$

Total penalty paid for 30 days $= S_{30}$

$= \frac{\cancel{30}^{15}}{\cancel{2}_1}[2 \times 200 + 29 \times 50]$

$= 15(400 + 1450)$

$= 15 \times 1850$

$= 27750$

$\therefore$ Total penalty = ₹ 27750

16. Let the value of first prize be ₹ x

Value of second prize = ₹ $(x-20)$

Value of third prize = ₹ $(x-40)$

Value of fourth prize = ₹ $(x-60)$

— — — — — — — — — —

— — — — — — — — — — .

The values of prizes form an A.P. with $a = x$ and $d = -20$ as the value of each prize decreases by ₹ 20

Total sum = ₹ 700

$\therefore \quad S_7 = 700$

or $\frac{7}{2}\left[2x + 6(-20)\right] = 700$

or $\frac{7}{\cancel{2}_1} \times \cancel{2}^1 (x - 60) = 700$

or $x - 60 = \frac{\cancel{700}^{100}}{\cancel{7}_1}$

or $x = 100 + 60$

or $x = 160$

$\therefore$ Value of first prize = ₹ 160

Value of second prize = 160 − 20 = ₹ 140

Value of third prize = 140 − 20 = ₹ 120

Value of fourth prize = 120 − 20 = ₹ 100

Value of fifth prize = 100 − 20 = ₹ 80

Value of sixth prize = 80 − 20 = ₹ 60

Value of seventh prize = 60 − 20 = ₹ 40

17. Number of trees planted by class I $= 1 \times 3 = 3$
Number of trees planted by class II $= 2 \times 3 = 6$
Number of trees planted by class III $= 3 \times 3 = 9$
Number of trees planted by class IV $= 4 \times 3 = 12$
— — — — — — —
— — — — — — — — —

$a_2 - a_1 = 6 - 3 = 3$

$a_3 - a_2 = 9 - 6 = 3$

$a_4 - a_3 = 12 - 9 = 3$

— — — — — — — — —
— — — — — —

Since $a_{k+1} - a_k$ is same for all the values of k
$\therefore 3, 6, 9, 12, ----$ form an A.P. with $a = 3$ and $d = 3$
Number of trees planted $= S_{12}$

$$= \frac{\cancelto{6}{12}}{\cancel{2}}\left[2 \times 3 + 11 \times 3\right]$$

$$= 6\,(6 + 33)$$

$$= 6 \times 39$$

$$= 234$$

18. Length of first semicircle, $l_1 = \pi \times 0.5 = 0.5\pi$ cm
Length of second semicircle, $l_2 = \pi \times 1 = \pi$ cm
Length of third semicircle, $l_3 = \pi \times 1.5 = 1.5\pi$ cm
Length of fourth semicircle, $l_4 = \pi \times 2 = 2\pi$ cm
— — — — — — — — — — —
— — — — — — — — — —

$a_2 - a_1 = \pi - 0.5\pi = 0.5\pi$ cm

$a_3 - a_2 = 1.5\pi - 0.5\pi = 0.5\pi$ cm

$a_4 - a_3 = 2\pi - 1.5\pi = 0.5\pi$ cm

— — — — — — — —
— — — — — — —

Since $a_{k+1} - a_k$ is same for all the values of k.
$\therefore 0.5\pi, \pi, 1.5\pi, 2\pi, ----$ form an A.P. with $a = 0.5\pi$

and $d = 0.5\pi$.

$\therefore$ Total length of spiral $= S_{13}$

$$= \frac{13}{2}(2 \times 0.5\pi + 12 \times 0.5\pi)$$

$$= \frac{13}{2}(\pi + 6\pi)$$

$$= \frac{13}{2} \times 7\pi$$

$$= \frac{13}{2} \times \cancel{7}^{1} \times \frac{\cancel{22}^{11}}{\cancel{7}_{1}}$$

$$= 143 \text{ cm}$$

19. No. of logs in bottom row $= 20$

No. of logs in next row $= 19$

No. of logs in next row $= 18$

No. of logs in next row $= 17$

— — — — — — — —

— — — — — — — —

$a_2 - a_1 = 19 - 20 = -1$

$a_3 - a_2 = 18 - 19 = -1$

$a_4 - a_3 = 17 - 18 = -1$

— — — — — — — —

— — — — — — — —

Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 20, 19, 18, 17, - - - - form an A.P. with $a = 20$, $d = -1$.

Let no. of rows in which 200 logs are placed be n.

$\therefore S_n = 200$

or $\frac{n}{2}[2 \times 20 + (n-1)(-1)] = 200$

or $n(40 - n + 1) = 400$

or $n(41 - n) = 400$

or $41n - n^2 = 400$

or $0 = n^2 - 41n + 400$

or $n^2 - 25n - 16n + 400 = 0$

or $n(n-25) - 16(n-25) = 0$

or $(n-25)(n-16) = 0$

either $n - 25 = 0$ or $n - 16 = 0$

or $n = 25$ or $n = 16$

When $n = 25$

No. of logs in top row $= a_{25}$

$= a + 24d$

$= 20 + 24(-1)$

$= 20 - 24$

$= -4$

We reject $n = 25$, as no. of logs (-4) cannot be negative.

$\therefore$ No. of rows $= 16$

No. of logs in top row $= a_{16}$

$= a + 15d$

$= 20 + 15(-1)$

$= 20 - 15$

$= 5$

20\. Distance covered to pick first potato $= 2 \times 5 = 10\,m$

Distance covered to pick second potato $= 2 \times (5+3) = 16\,m$

Distance covered to pick third potato $= 2 \times (5 + 3 \times 2) = 22\,m$

Distance covered to pick fourth potato $= 2 \times (5 + 3 \times 3) = 28\,m$

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$a_2 - a_1 = 16 - 10 = 6\,m$

$a_3 - a_2 = 22 - 16 = 6\,m$

$a_4 - a_3 = 28 - 22 = 6$ m

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Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 10, 16, 22, 28, ---- form an A.P. with $a = 10$ and $d = 6$

$\therefore$ Total distance covered $= S_{10}$

$$= \frac{\cancel{10}^{5}}{\cancel{2}_{1}}[2 \times 10 + 9 \times 6]$$

$$= 5\ (20 + 54)$$

$$= 5 \times 74$$

$$= 370 \text{ m}$$