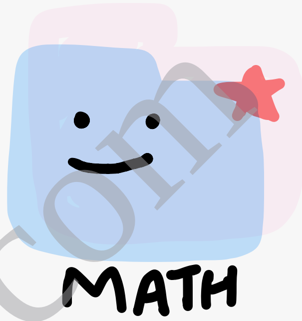
Arithmetic Progressions
Ex. 5.2



+ ×

− ÷

Ex. 5.2

1 (i)

$a = 7$

$d = 3$

$n = 8$

$a_n = ?$

$a_8 = a + (8-1)d$

$= 7 + 7 \times 3$

$= 7 + 21$

$= 28$

(ii)

$a = -18$

$n = 10$

$d = ?$

$a_{10} = 0$

or $a + (10-1)d = 0$

or $-18 + 9d = 0$

or $9d = 18$

or $d = \frac{\cancel{18}^{\,2}}{\cancel{9}_{\,1}}$

or $d = 2$

(iii)

$d = -3$

$n = 18$

$a_n = -5$

$a_{18} = -5$

or $a + (18-1)d = -5$

or $a + 17(-3) = -5$

or $a - 51 = -5$

or $a = 51 - 5$

or $a = 46$

(iv) $a = -18.9$

$d = 2.5$

$a_n = 3.6$

$n = ?$

$a_n = 3.6$

or $a + (n-1)d = 3.6$

or $-18.9 + (n-1)2.5 = 3.6$

or $(n-1)\,2.5 = 3.6 + 18.9$

or $(n-1)\,2.5 = 22.5$

or $n - 1 = \frac{22.5}{2.5}$

or $n - 1 = 9$

or $n = 9 + 1$

or $n = 10$

(v) $a = 3.5$

$d = 0$

$n = 105$

$a_n = a + (n-1)d$

$a_{105} = 3.5 + (105-1)0$

$= 3.5 + 0$

$= 3.5$

20(i) $10, 7, 4, ----$

$a = 10$

$d = 7 - 10 = -3$

$a_n = a + (n-1)d$

or $a_{30} = 10 + (30-1)(-3)$

$= 10 + 29(-3)$

or $a_{30} = 10 - 87$
or $a_{30} = -77$
Ans. - (C)

(11) $-3, -\frac{1}{2}, 2, ----$

$a = -3$
$d = -\frac{1}{2} - (-3)$

or $d = -\frac{1}{2} + 3$

or $d = \frac{5}{2}$

$a_n = a + (n-1)d$
or $a_{11} = -3 + (11-1)\frac{5}{2}$

$= -3 + \cancel{10}^{5} \times \frac{5}{\cancel{2}_{1}}$

$= -3 + 25$
$= 22$
Ans. - (B)

3(1) $a = 2$ — (1)
$a_3 = 26$
or $a + 2d = 26$
or $2 + 2d = 26$ (using equation (1))
or $2d = 26 - 2$
or $2d = 24$
or $d = \frac{\cancel{24}^{12}}{\cancel{2}_{1}}$

or $d = 12$

$a_2 = a + d$

$= 2 + 12$

$= 14$

(ii) $a_2 = 13 \Rightarrow a + d = 13$ — (i)

$a_4 = 3 \Rightarrow a + 3d = 3$ — (ii)

Subtracting equation (i) from (ii)

$a + 3d - (a + d) = 3 - 13$

or $\cancel{a} + 3d - \cancel{a} - d = -10$

or $2d = -10$

or $d = -\frac{\cancel{10}^{5}}{\cancel{2}_{1}}$

or $d = -5$

Put the value of d in equation (i)

$a - 5 = 13$

or $a = 13 + 5$

or $a = 18$

$a_3 = a + 2d$

$= 18 + 2(-5)$

$= 18 - 10$

$= 8$

(iii) $a = 5$ — (i)

$a_4 = 9\frac{1}{2}$

or $a_4 = \frac{19}{2}$

or $a + 3d = \frac{19}{2}$

or $5 + 3d = \frac{19}{2}$ (using equation ①)

or $3d = \frac{19}{2} - 5$

or $3d = \frac{19-10}{2}$

or $3d = \frac{9}{2}$

or $d = \frac{\cancel{9}\,3}{2 \times \cancel{3}\,1}$

or $d = \frac{3}{2}$

$a_2 = a + d$

or $a_2 = 5 + \frac{3}{2}$

or $a_2 = \frac{13}{2}$ or $a_2 = 6\frac{1}{2}$

$a_3 = a + 2d$

or $a_3 = 5 + \cancel{2}^{1} \times \frac{3}{\cancel{2}_{1}}$

or $a_3 = 5 + 3$

or $a_3 = 8$

(iv) $a = -4$ —— ①

$a_6 = 6$

or $a + 5d = 6$

or $-4 + 5d = 6$ (using equation ①)

or $5d = 6 + 4$

or $5d = 10$

or $d = \frac{\cancel{10}^{2}}{\cancel{5}_{1}}$

or $d = 2$

$a_2 = a + d$

or $a_2 = -4 + 2$

or $a_2 = -2$

$a_3 = a + 2d$

or $a_3 = -4 + 2 \times 2$

or $a_3 = -4 + 4$

or $a_3 = 0$

$a_4 = a + 3d$

or $a_4 = -4 + 3 \times 2$

or $a_4 = -4 + 6$

or $a_4 = 2$

$a_5 = a + 4d$

or $a_5 = -4 + 4 \times 2$

or $a_5 = -4 + 8$

or $a_5 = 4$

(v) $a_2 = 38 \Rightarrow a + d = 38$ — (i)

$a_6 = -22 \Rightarrow a + 5d = -22$ — (ii)

Subtracting equation (i) from (ii)

$a + 5d - (a + d) = -22 - 38$

or $\cancel{a} + 5d - \cancel{a} - d = -60$

or $4d = -60$

or $d = -\frac{\cancel{60}^{15}}{\cancel{4}_{1}}$

or $d = -15$

Put the value of d in equation ①

$a + (-15) = 38$

or $a = 38 + 15$

or $a = 53$

$a_3 = a + 2d$

or $a_3 = 53 + 2(-15)$

or $a_3 = 53 - 30$

or $a_3 = 23$

$a_4 = a + 3d$

or $a_4 = 53 + 3(-15)$

or $a_4 = 53 - 45$

or $a_4 = 8$

$a_5 = a + 4d$

or $a_5 = 53 + 4(-15)$

or $a_5 = 53 - 60$

or $a_5 = -7$

4. 3, 8, 13, 18, -----

$a = 3$

$d = 8 - 3 = 5$

Let the n^{th} term be 78

$a_n = 78$

or $a + (n-1)d = 78$

or $3 + (n-1)5 = 78$

or $(n-1)5 = 78 - 3$

or $(n-1)5 = 75$

or $n-1 = \frac{\cancel{75}\,15}{\cancel{5}\,1}$

or $n = 15+1$

or $n = 16$

$\therefore$ 16^{th} term is 78.

5(i) 7, 13, 19, ----, 205

$a = 7$

$d = 13-7 = 6$

Let the number of terms be n.

$\therefore a_n = 205$

or $a + (n-1)d = 205$

or $7 + (n-1)6 = 205$

or $(n-1)6 = 205-7$

or $(n-1)6 = 198$

or $n-1 = \frac{\cancel{198}\,33}{\cancel{6}\,1}$

or $n-1 = 33$

or $n = 33+1$

or $n = 34$

$\therefore$ Number of terms $= 34$

(ii) $18, 15\frac{1}{2}, 13, ----, (-47)$

$a = 18$

$d = 15\frac{1}{2} - 18$

or $d = \frac{31}{2} - 18$

or $d = \frac{31-36}{2}$

or $d = -\frac{5}{2}$

Let the number of terms be n.

$\therefore a_n = -47$

or $a+(n-1)d = -47$

or $18 + (n-1)\left(-\frac{5}{2}\right) = -47$

or $(n-1)\left(-\frac{5}{2}\right) = -47-18$

or $(n-1)\left(-\frac{5}{2}\right) = -65$

or $n-1 = -\cancel{65}^{13} \times \left(-\frac{2}{\cancel{5}_1}\right)$

or $n-1 = 26$

or $n = 27$

$\therefore$ Number of terms $= 27$

16. $11, 8, 5, 2, \ldots$

$a = 11$

$d = 8-11 = -3$

Let the n^{th} term be (-150).

$\therefore a_n = -150$

or $a+(n-1)d = -150$

or $11+(n-1)(-3) = -150$

or $(n-1)(-3) = -150-11$

or $n-1 = \frac{-161}{-3}$

or $n = \frac{161}{3} + 1$

or $n = \frac{161+3}{3}$

or $n = \frac{164}{3}$

Since n is not a positive integer.
$\therefore (-150)$ is not a term of the given A.P.

7. Let 'a' be the first term and 'd' be the common difference
$\therefore a_{11} = 38 \Rightarrow a + 10d = 38$ — (i)
and $a_{16} = 73 \Rightarrow a + 15d = 73$ — (ii)
Subtracting equation (i) from equation (ii)
$a + 15d - (a + 10d) = 73 - 38$
or $\cancel{a} + 15d - \cancel{a} - 10d = 35$
or $5d = 35$
or $d = \frac{\cancel{35}\,7}{\cancel{5}\,1}$

or $d = 7$
Put the value of d in equation (i)
$a + 10 \times 7 = 38$
or $a + 70 = 38$
or $a = 38 - 70$
or $a = -32$
$\therefore a_{31} = a + 30d$
$= -32 + 30 \times 7$
$= -32 + 210$
$= 178$

8. Let 'a' be the first term and 'd' be the common difference.

$n = 50$ (given)

$a_3 = 12 \Rightarrow a + 2d = 12$ — (i)

$a_{50} = 106 \Rightarrow a + 49d = 106$ — (ii)

Subtracting equation (i) from equation (ii)

$a + 49d - (a + 2d) = 106 - 12$

or $\cancel{a} + 49d - \cancel{a} - 2d = 94$

or $47d = 94$

or $d = \frac{\cancel{94}\,2}{\cancel{47}\,1}$

or $d = 2$

Put the value of d in equation (i)

$a + 2 \times 2 = 12$

or $a + 4 = 12$

or $a = 12 - 4$

or $a = 8$

$\therefore a_{29} = a + 28d$

$= 8 + 28 \times 2$

$= 8 + 56$

$= 64$

$\therefore 29^{th}$ term is 64.

9. Let 'a' be the first term and 'd' be the common difference.

$\therefore a_3 = 4 \Rightarrow a + 2d = 4$ — (i)

and $a_9 = -8 \Rightarrow a + 8d = -8$ — (ii)

Subtracting equation (i) from equation (ii)

$a + 8d - (a + 2d) = -8 - 4$

or $\cancel{a}+8d-\cancel{a}-2d=-12$

or $6d=-12$

or $d=-\frac{\cancel{12}\,2}{\cancel{6}\,1}$

or $d=-2$

Put the value of d in equation ①

$a+2(-2)=4$

or $a-4=4$

or $a=4+4$

or $a=8$

Let the n^{th} term be zero.

$\therefore\ a_n=0$

or $a+(n-1)d=0$

or $8+(n-1)(-2)=0$

or $(n-1)(-2)=-8$

or $n-1=\frac{-\cancel{8}\,4}{-\cancel{2}\,1}$

or $n=4+1$

or $n=5$

$\therefore\ 5^{th}$ term is zero.

10. Let 'a' be the first term and 'd' be the common difference.

$\therefore\ a_{17}=a_{10}+7$

or $a+16d=a+9d+7$

or $\cancel{a}+16d-\cancel{a}-9d=7$

or $7d=7$

or $d=\frac{\cancel{7}\,1}{\cancel{7}\,1}$

or $d = 1$

$\therefore$ Common difference $= 1$

11. $3, 15, 27, 39, \text{------}$

$a = 3$

$d = 15 - 3 = 12$

Let the n^{th} term be 132 more than the 54^{th} term.

$\therefore a_n = a_{54} + 132$

or $a + (n-1)d = a + 53d + 132$

or $\cancel{a} + (n-1)12 = \cancel{a} + 53 \times 12 + 132$

Dividing both sides by 12

$n - 1 = 53 + 11$

or $n - 1 = 64$

or $n = 64 + 1$

or $n = 65$

$\therefore 65^{th}$ term is 132 more than the 54^{th} term.

12. Let the first terms of the two A.P.'s be 'A' and 'a' and 'd' be the common difference.

$\therefore A_{100} - a_{100} = 100$

or $A + 99d - (a + 99d) = 100$

or $A + \cancel{99d} - a - \cancel{99d} = 100$

or $A - a = 100$ —— ①

$A_{1000} - a_{1000}$

$= A + 999d - (a + 999d)$

$= A + \cancel{999d} - a - \cancel{999d}$

$= A - a$

$= 100$ (using equation ①)

$\therefore$ The difference between the 1000^{th} terms is 100.

13. Three-digit numbers divisible by 7 are

105, 112, 119, 126, ----, 994

$a_2 - a_1 = 112 - 105 = 7$

$a_3 - a_2 = 119 - 112 = 7$

$a_4 - a_3 = 126 - 119 = 7$

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Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 105, 112, 119, 126, ----, 994 form an A.P. with $a = 105$ and $d = 7$

Let $a_n = 994$

$\therefore \quad a + (n-1)d = 994$

or $\quad 105 + (n-1)7 = 994$

or $\quad (n-1)7 = 994 - 105$

or $\quad (n-1)7 = 889$

or $\quad n - 1 = \frac{\cancel{889}^{127}}{\cancel{7}_1}$

or $\quad n = 127 + 1$

or $\quad n = 128$

$\therefore$ 128 three-digit numbers are divisible by 7.

14. Multiples of 4 lying between 10 and 250 are

12, 16, 20, 24, 28, -----, 248

$a_2 - a_1 = 16 - 12 = 4$

$a_3 - a_2 = 20 - 16 = 4$

$a_4 - a_3 = 24 - 20 = 4$

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Since $a_{k+1} - a_k$ is same for all the values of k.

$\therefore$ 12, 16, 20, 24, ― ― ― ―, 248 form an A.P. with $a = 12$ and $d = 16$

Let $a_n = 248$

or $a + (n-1)d = 248$

or $12 + (n-1)4 = 248$

or $(n-1)4 = 248 - 12$

or $n - 1 = \frac{\cancel{236}\ 59}{\cancel{4}\ 1}$

or $n = 59 + 1$

or $n = 60$

$\therefore$ 60 multiples of 4 lie between 10 and 250.

15. 63, 65, 67, ― ― ― ― ―

$a = 63$

$d = 65 - 63 = 2$

3, 10, 17, ― ― ― ― ―

$A = 3$

$D = 10 - 3 = 7$

$a_n = A_n$ (given)

or $a + (n-1)d = A + (n-1)D$

or $63 + (n-1)2 = 3 + (n-1)7$

or $63 - 3 = (n-1)7 - (n-1)2$

or $60 = (n-1)(7-2)$

or $60 = (n-1)5$

or $n - 1 = \frac{\cancel{60}\,12}{\cancel{5}\,1}$

or $n = 12 + 1$

or $n = 13$

$\therefore$ For $n = 13$, the n^{th} terms of two given A.P.'s are equal

16. Let 'a' be the first term and 'd' be the common difference.

$\therefore$ $a_3 = 16 \Rightarrow a + 2d = 16$ — (1)

and $a_7 = a_5 + 12$

or $a + 6d = a + 4d + 12$

or $\cancel{a} + 6d - \cancel{a} - 4d = 12$

or $2d = 12$

or $d = \frac{\cancel{12}\,6}{\cancel{2}\,1}$

or $d = 6$

Put the value of d in equation (1)

$a + 2 \times 6 = 16$

or $a + 12 = 16$

or $a = 16 - 12$

or $a = 4$

$\therefore$ $a_1 = a = 4$

$a_2 = a + d = 4 + 6 = 10$

$a_3 = a + 2d = 4 + 2 \times 6 = 16$

$a_4 = a + 3d = 4 + 3 \times 6 = 22$

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$\therefore$ A.P. is 4, 10, 16, 22, - - - - - -

17. $3, 8, 13, \text{-----}, 253$

$a = 3$

$d = 8-3 = 5$, $l = 253$

n^{th} term from the end $= l-(n-1)d$

$\therefore 20^{th}$ term from the end $= 253-(20-1)5$

$= 253 - 19 \times 5$

$= 253 - 95$

$= 158$

18. Let 'a' be the first term and 'd' be the common difference.

$a_4 + a_8 = 24$

or $a + 3d + a + 7d = 24$

or $2a + 10d = 24$

Dividing both sides by 2

$a + 5d = 12$ — (i)

$a_6 + a_{10} = 44$

or $a + 5d + a + 9d = 44$

or $2a + 14d = 44$

Dividing both sides by 2

$a + 7d = 22$ — (ii)

Subtracting equation (i) from equation (ii)

$a + 7d - (a + 5d) = 22 - 12$

or $\cancel{a} + 7d - \cancel{a} - 5d = 10$

or $2d = 10$

or $d = \frac{\cancel{10}^{5}}{\cancel{2}_{1}}$

or $d = 5$

Put the value of d in equation (1)

$a + 5 \times 5 = 12$

or $a + 25 = 12$

or $a = 12 - 25$

or $a = -13$

$\therefore a_1 = a = -13$

$a_2 = a + d = -13 + 5 = -8$

$a_3 = a + 2d = -13 + 2 \times 5 = -13 + 10 = -3$

$\therefore$ First three terms of the A.P. are $-13, -8, -3, \ldots\ldots$

19. Salary in 1995 = ₹ 5000

Salary in 1996 = 5000 + 200 = ₹ 5200

Salary in 1997 = 5200 + 200 = ₹ 5400

Salary in 1999 = 5400 + 200 = ₹ 5600

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$a_2 - a_1 = 5200 - 5000 = 200$

$a_3 - a_2 = 5400 - 5200 = 200$

$a_4 - a_3 = 5600 - 5400 = 200$

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Since $a_{k+1} - a_k$ is same for all values of k.

$\therefore$ 5000, 5200, 5400, 5600, ----- form an A.P. with

$a = 5000$ and $d = 200$.

Let the income be ₹ 7000 in the n^{th} week.

$\therefore a_n = 7000$

or $a + (n-1)d = 7000$

or $5000 + (n-1)200 = 7000$

or $(n-1)200 = 7000 - 5000$

or $(n-1)\,200 = 2000$

or $n - 1 = \frac{\cancel{2000}^{10}}{\cancel{200}_{1}}$

or $n-1=10$

or $n=10+1$

or $n=11$

$\therefore$ The income is ₹ 7000 in the year 2005(1995+11−1)

20. Saving in first week = ₹ 5

Saving in second week = 5 + 1.75 = ₹ 6.75

Saving in third week = 6.75 + 1.75 = ₹ 8.50

Saving in fourth week = 8.50 + 1.75 = ₹ 10.25

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$a_2 - a_1 = 6.75 - 5 = 1.75$

$a_3 - a_2 = 8.50 - 6.75 = 1.75$

$a_4 - a_3 = 10.25 - 8.50 = 1.75$

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Since $a_{k+1} - a_k$ is same for all values of k.

$\therefore$ 5, 6.75, 8.50, 10.25, — — — — form an A.P. with $a=5$ and $d=1.75$

Saving in the n^{th} week = ₹ 20.75

$\therefore \quad a_n = 20.75$

or $\quad a+(n-1)d = 20.75$

or $\quad 5+(n-1)1.75 = 20.75$

or $\quad (n-1)1.75 = 20.75-5$

or $\quad (n-1)1.75 = 15.75$

or $\quad n-1 = \dfrac{15.75}{1.75}$

or $\quad n-1 = 9$

or $\quad n = 9+1$

or $\quad n = 10$

$\therefore$ Saving becomes ₹ 20.75 in the 10^{th} week.