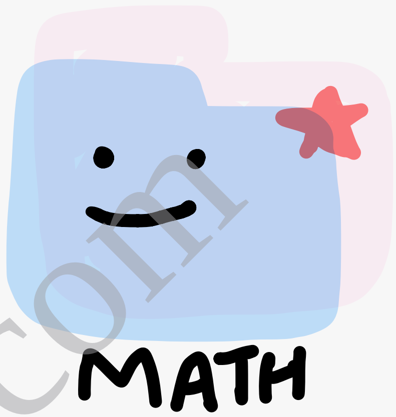
Polynomials
Ex. 2.5



$+ \quad \times$

$- \quad \div$

Ex. 2.5

1 (i) $(x+4)(x+10)$

$= x^2+(4+10)x+4\times 10 \quad [(x+a)(x+b) = x^2+(a+b)x+ab]$

$= x^2+14x+40$

(ii) $(x+8)(x-10)$

$= (x+8)[x+(-10)]$

$= x^2+[8+(-10)]x+8(-10) \quad [(x+a)(x+b) = x^2+(a+b)x+ab]$

$= x^2-2x-80$

(iii) $(3x+4)(3x-5)$

$= (3x+4)[3x+(-5)]$

$= (3x)^2+[4+(-5)]3x+4(-5) \quad [(x+a)(x+b) = x^2+(a+b)x+ab]$

$= 9x^2-3x-20$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$= (y^2)^2-\left(\frac{3}{2}\right)^2 \quad [(a+b)(a-b) = a^2-b^2]$

$= y^4-\frac{9}{4}$

(v) $(3-2x)(3+2x)$

$= (3)^2-(2x)^2 \quad [(a+b)(a-b) = a^2-b^2]$

$= 9-4x^2$

2 (i) 103×107

$= (100+3)(100+7)$

$= (100)^2+(3+7)100+3\times 7 \quad [(x+a)(x+b) = x^2+(a+b)x+ab]$

$= 10000+1000+21$

$= 11021$

(ii) 95×96

$= (100-5)(100-4)$

$= [100+(-5)][100+(-4)]$ $[(x+a)(x+b) = x^2+(a+b)x+ab]$

$= (100)^2 + [(-5)+(-4)]100 + (-5)(-4)$

$= 10000 - 900 + 20$

$= 9120$

(iii) 104×96

$= (100+4)(100-4)$

$= (100)^2 - (4)^2$ $[(a+b)(a-b) = a^2 - b^2]$

$= 10000 - 16$

$= 9984$

3(i) $9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + y^2$

$= (3x+y)^2$ $[a^2 + 2ab + b^2 = (a+b)^2]$

$= (3x+y)(3x+y)$

(ii) $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y-1)^2$ $[a^2 - 2ab + b^2 = (a-b)^2]$

$= (2y-1)(2y-1)$

(iii) $x^2 - \frac{y^2}{100}$

$= (x)^2 - \left(\frac{y}{10}\right)^2$

$= \left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$ $[\because a^2-b^2=(a+b)(a-b)]$

4(i) $(x+2y+4z)^2$

$= x^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(4z)(x)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

$= [2x+(-y)+z]^2$

$= (2x)^2+(-y)^2+z^2+2(2x)(-y)+2(-y)(z)+2(z)(2x)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

$= (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a-7b-c)^2$

$= [3a+(-7b)^2+(-c)]^2$

$= (3a)^2+(-7b)^2+(-c)^2+2(3a)(-7b)+2(-7b)(-c)+2(-c)(3a)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= 9a^2+49b^2+c^2-42ab+14bc-6ac$

(v) $(-2x+5y-3z)^2$

$= [-2x+5y+(-3z)]^2$

$= (-2x)^2+(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)(-3z)+2(-3z)(-2x)$

$[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca]$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

$= \left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right]^2$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + \overset{1}{\cancel{2}}\left(\frac{1}{4}a\right)\left(-\frac{1}{\cancel{2}_{1}}b\right) + \overset{1}{\cancel{2}}\left(-\frac{1}{\cancel{2}_{1}}b\right)(1) + \overset{1}{\cancel{2}}(1)\left(\frac{1}{\cancel{4}_{2}}a\right)$

$\left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\right]$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

5(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$\left[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2\right]$

$= [2x + 3y + (-4z)]^2$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (-\sqrt{2}x)^2 + y^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$\left[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2\right]$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

6(i) $(2x+1)^3$

$\left[(a+b)^3 = a^3 + 3ab(a+b) + b^3\right]$

$= (2x)^3 + 3(2x)(1)(2x+1) + (1)^3$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3$

$[(x-y)^3 = x^3 - 3xy(x-y) - y^3]$

$= (2a)^3 - 3(2a)(3b)(2a-3b) - (3b)^3$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $\left[\frac{3}{2}x+1\right]^3$

$[(a+b)^3 = a^3 + 3ab(a+b) + b^3]$

$= \left(\frac{3}{2}x\right)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right) + (1)^3$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $\left(x - \frac{2}{3}y\right)^3$

$[(a-b)^3 = a^3 - 3ab(a-b) - b^3]$

$= (x)^3 - \overset{1}{\cancel{3}}(x)\left(\frac{2}{\cancel{3}_1}y\right)\left(x - \frac{2}{3}y\right) - \left(\frac{2}{3}y\right)^3$

$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$

7(i) $(99)^3$

$= (100-1)^3$

$(a-b)^3 = a^3 - 3ab(a-b) - b^3$

$= (100)^3 - 3(100)(1)(100-1) - (1)^3$

$= 1\,000000 - 30000 + 300 - 1$

$= 1000300 - 30001$

$= 970299$

(ii) $(102)^3$

$= (100+2)^3$

$[(a+b)^3 = a^3 + 3ab(a+b) + b^3]$

$= (100)^3 + 3(100)(2)(100+2) + (2)^3$

$= 1000000 + 60000 + 1200 + 8$

$= 1061208$

(iii) 998^3

$= (1000-2)^3$

$(a-b)^3 = a^3 - 3ab(a-b) - b^3$

$= (1000)^3 - 3(1000)(2)(1000-2) - (2)^3$

$= 1000000000 - 6000000 + 12000 - 8$

$= 1000012000 - 6000008$

$= 994011992$

8(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)(b)(2a+b)$

$= (2a+b)^3 \quad [x^3 + y^3 + 3xy(x+y) = (x+y)^3]$

$= (2a+b)(2a+b)(2a+b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)(b)(2a-b)$

$= (2a-b)^3 \quad [x^3 - y^3 - 3xy(x-y) = (x-y)^3]$

$= (2a-b)(2a-b)(2a-b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)(5a)(3-5a)$

$= (3-5a)^3 \quad [x^3 - y^3 - 3xy(x-y) = (x-y)^3]$

$= (3-5a)(3-5a)(3-5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3 \quad [x^3 - y^3 - 3xy(x-y) = (x-y)^3]$

$= (4a - 3b)(4a - 3b)(4a - 3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3 \quad [x^3 - y^3 - 3xy(x-y) = (x-y)^3]$

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

9(i) Verify $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

Let $p(x, y) = x^3 + y^3$

Put $x = -y$

$\therefore p(-y, y) = (-y)^3 + y^3$

$= -y^3 + y^3$

$= 0$

$\therefore$ by factor theorem, $(x+y)$ is a factor of $p(x,y)$

$$
\begin{array}{r|l|l}
x+y & \cancel{x^3} + y^3 & x^2 - xy + y^2 \\
 & \cancel{x^3} + x^2y & \\
 & - \quad\quad - & \\ \hline
 & -\cancel{x^2y} + y^3 & \\
 & -\cancel{x^2y} - xy^2 & \\
 & + \quad\quad + & \\ \hline
 & \cancel{y^3} + \cancel{xy^2} & \\
 & \cancel{y^3} + \cancel{xy^2} & \\
 & - \quad\quad - & \\ \hline
 & 0 &
\end{array}
$$

$\therefore x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

Hence verified.

(ii) $x^3 - y^3 = (x-y)(x^2+xy+y^2)$

Let $p(x,y) = x^3 - y^3$

Put $x = y$

$p(y,y) = y^3 - y^3$

$= 0$

$\therefore$ by factor theorem, $(x-y)$ is a factor of $p(x,y)$

$$x-y \,\big)\, x^3 - y^3 \,\big(\, x^2 + xy + y^2$$

$$\cancel{x^3} \quad -x^2y$$

$$- \quad +$$

$$\cancel{x^2y} - y^3$$

$$\cancel{x^2y} \quad - xy^2$$

$$- \quad +$$

$$\cancel{xy^2} - \cancel{y^3}$$

$$\cancel{xy^2} - \cancel{y^3}$$

$$- \quad +$$

$$0$$

$\therefore x^3 - y^3 = (x-y)(x^2+xy+y^2)$ Hence verified.

10 (i) $27y^3 + 125z^3$

$= (3y)^3 + (5z)^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= (3y+5z)\left[(3y)^2 - (3y)(5z) + (5z)^2\right]$

$= (3y+5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= (4m-7n)\left[(4m)^2 + (4m)(7n) + (7n)^2\right]$

$= (4m-7n)(16m^2 + 28mn + 49n^2)$

11. $27x^3+y^3+z^3-9xyz$

$= (3x)^3+y^3+z^3-3(3x)(y)(z)$

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= (3x+y+z)[(3x)^2+y^2+z^2-(3x)(y)-(y)(z)-(z)(3x)]$

$= (3x+y+z)\ (9x^2+y^2+z^2-3xy-yz-3xz)$

12. Verify: $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

L.H.S= $x^3+y^3+z^3-3xyz$

$=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Multiplying and dividing by 2

$= 2\times\frac{1}{2}(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$= \frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$

$= \frac{1}{2}(x+y+z)(x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx)$

$= \frac{1}{2}(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(z^2+x^2-2xz)]$

$= \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

= R.H.S.

Hence verified

13. $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

or $x^3+y^3+z^3-3xyz=0\,(x^2+y^2+z^2-xy-yz-zx)$ [$\because x+y+z=0$ given]

or $x^3+y^3+z^3-3xyz=0$

or $x^3+y^3+z^3=3xyz$

Hence proved

14(i) $(-12)^3+(7)^3+(5)^3$

Here $x=-12$, $y=7$, $z=5$

$x+y+z=-12+7+5=-12+12=0$

If $x+y+z=0$, then $x^3+y^3+z^3=3xyz$

$\therefore (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$

$=-1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Here $x=28$, $y=-15$, $z=-13$

$x+y+z=28+(-15)+(-13)$

$=28-28$

$=0$

If $x+y+z=0$, then $x^3+y^3+z^3=3xyz$

$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

$=16380$

15(i) Area $=25a^2-35a+12$

$=25a^2-20a-15a+12$

$=5a(5a-4)-3(5a-4)$

$=(5a-4)(5a-3)$

$\therefore$ length $=(5a-4)$ units

breadth $=(5a-3)$ units

(ii) Area $=35y^2+13y-12$

$=35y^2+28y-15y-12$

$=7y(5y+4)-3(5y+4)$

$=(5y+4)(7y-3)$

$\therefore$ length $=(5y+4)$ units

breadth $=(7y-3)$ units

10) Volume $= 3x^2 - 12x$

$= 3x(x-4)$

$\therefore$ length $= 3$ units

breadth $= x$ units

height $= (x-4)$ units

11) Volume $= 12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 - 3y + 5y - 5)$

$= 4k[3y(y-1) + 5(y-1)]$

$= 4k(y-1)(3y+5)$

$\therefore$ length $= 4k$ units

breadth $= (y-1)$ units

height $= (3y+5)$ units