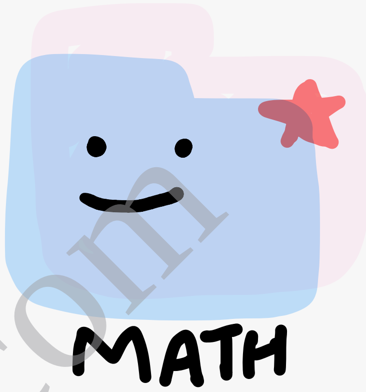
Polynomials
Ex. 2.4



$+ \quad \times$

$- \quad \div$

Ex. 2.4

1. (i) Let $p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0$

$\therefore$ by factor theorem

$(x+1)$ is a factor of $p(x)$.

(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + 1$

$= 1 \neq 0$

Since remainder $\neq 0$

$\therefore$ $(x+1)$ is not a factor of $p(x)$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= -1 - \cancel{3} + \cancel{3} - \cancel{1} + \cancel{1}$

$= -1 \neq 0$

Since remainder $\neq 0$

$\therefore$ $(x+1)$ is not a factor of $p(x)$.

(iv) Let $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 - \cancel{\sqrt{2}} + \cancel{\sqrt{2}}$

$= -2 + 2$

$= 0$

$\therefore$ by factor theorem

$(x+1)$ is a factor of $p(x)$.

2(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1$

$= 3 - 3$

$= 0$

Since remainder $= 0$

$\therefore$ $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= 13 - 14$

$= -1 \neq 0$

Since remainder $\neq 0$

$\therefore$ $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

$p(3) = 3^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9$

$= 36 - 36$

$= 0$

Since remainder $= 0$

$\therefore$ $g(x)$ is a factor of $p(x)$.

3.(i) $p(x) = x^2 + x + k$

Since $(x - 1)$ is a factor of $p(x)$

$\therefore$ by factor th^m

$p(1) = 0$

or $1^2 + 1 + k = 0$

or $1+1+k=0$

or $k+2=0$

or $k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

$(x-1)$ is a factor of $p(x)$

$\therefore$ by factor theorem

$p(1)=0$

or $2(1)^2+k(1)+\sqrt{2}=0$

or $2+k+\sqrt{2}=0$

or $k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

$(x-1)$ is a factor of $p(x)$

$\therefore$ by factor theorem

$p(1)=0$

or $k(1)^2-\sqrt{2}(1)+1=0$

or $k-\sqrt{2}+1=0$

or $k=\sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

$(x-1)$ is a factor of $p(x)$

$\therefore$ by factor theorem

$p(1)=0$

or $k(1)^2-3(1)+k=0$

or $k-3+k=0$

or $2k=3$

or $k=\frac{3}{2}$

4) (i) $12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4$

$= 3x^2 + 3x - 4x - 4$

$= 3x(x + 1) - 4(x + 1)$

$= (x + 1)(3x - 4)$

5) Let $p(x) = x^3 - 2x^2 - x + 2$

Factors of 2 are $\pm 1, \pm 2$

Put $x = 1$

$p(1) = 1^3 - 2(1)^2 - 1 + 2$

$= 1 - 2 - 1 + 2$

$= 3 - 3$

$= 0$

$\therefore$ by factor theorem

$(x - 1)$ is a factor of $p(x)$.

$$\begin{array}{r|l} & x^2 - x - 2 \\ x-1 & \cancel{x^3} - 2x^2 - x + 2 \\ & \cancel{x^3} - x^2 \\ & - \quad + \\ \hline & -\cancel{x^2} - x + 2 \\ & -\cancel{x^2} + x \\ & + \quad - \\ \hline & -2\cancel{x} + \cancel{2} \\ & -\cancel{2x} + \cancel{2} \\ & + \quad - \\ \hline & 0 \end{array}$$

$\therefore\ p(x) = (x-1)\ (x^2 - x - 2)$

$= (x-1)\ (x^2 - 2x + x - 2)$

$= (x-1)\ [x(x-2) + 1(x-2)]$

$= (x-1)\ (x-2)\ (x+1)$

<u>OR</u>

$x^3 - 2x^2 - x + 2$

$= x^2(x-2) - 1(x-2)$

$= (x-2)\ (x^2 - 1^2)$

$= (x-2)\ (x+1)\ (x-1)$ $\quad [\because\ a^2 - b^2 = (a+b)(a-b)]$

(11) Let $p(x) = x^3 - 3x^2 - 9x - 5$

Factors of 5 are $\pm 1, \pm 5$

$p(1) = 1^3 - 3(1)^2 - 9(1) - 5$

$= 1 - 3 - 9 - 5$

$= 1 - 17$

$= -16 \neq 0$

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5$

$= 9 - 9$

$= 0$

$\therefore$ by factor theorem

$(x+1)$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\hline
x+1 & x^3 - 3x^2 - 9x - 5 \\
 & \cancel{x^3} + x^2 \\
 & - \quad - \\
\hline
 & -\cancel{4x^2} - 9x - 5 \\
 & -\cancel{4x^2} - 4x \\
 & + \quad + \\
\hline
 & -\cancel{5x} - \cancel{5} \\
 & -\cancel{5x} - \cancel{5} \\
 & + \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$\therefore p(x) = (x+1)(x^2 - 4x - 5)$

$= (x+1)(x^2 - 5x + x - 5)$

$= (x+1)[x(x-5) + 1(x-5)]$

$= (x+1)(x-5)(x+1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

Factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$p(1) = 1^3 + 13(1)^2 + 32(1) + 20$

$= 1 + 13 + 32 + 20$

$= 66 \neq 0$

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33$

$= 0$

$\therefore$ by factor theorem

$(x+1)$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & \cancel{x^3} + x^2 \\
 & - \quad - \\
\hline
 & 12x^2 + 32x + 20 \\
 & 12x^2 + 12x \\
 & - \quad - \\
\hline
 & 20x + 20 \\
 & 20x + 20 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

$\therefore p(x) = (x+1)\,(x^2 + 12x + 20)$

$= (x+1)\,(x^2 + 10x + 2x + 20)$

$= (x+1)\,[x(x+10) + 2(x+10)]$

$= (x+1)\,(x+10)\,(x+2)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

Factors of 1 are ± 1

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1$

$= 3 - 3$

$= 0$

$\therefore$ by factor theorem

$(y-1)$ is a factor of $p(y)$

$$\begin{array}{r l}
 & 2y^2 + 3y + 1 \\
y - 1 \,\big) & 2y^3 + y^2 - 2y - 1 \\
 & \cancel{2y^3} - 2y^2 \\
 & - \quad + \\ \hline
 & \cancel{3y^2} - 2y - 1 \\
 & \cancel{3y^2} - 3y \\
 & - \quad + \\ \hline
 & \cancel{y} - \cancel{1} \\
 & \cancel{y} - \cancel{1} \\
 & - \quad + \\ \hline
 & 0 \\ \hline
\end{array}$$

$$\begin{aligned}
\therefore\ p(y) &= (y-1)\,(2y^2 + 3y + 1) \\
&= (y-1)\,(2y^2 + 2y + y + 1) \\
&= (y-1)\,[2y(y+1) + 1(y+1)] \\
&= (y-1)\,(y+1)\,(2y+1)
\end{aligned}$$

OR

$$\begin{aligned}
& 2y^3 + y^2 - 2y - 1 \\
&= y^2(2y+1) - 1(2y+1) \\
&= (2y+1)(y^2 - 1^2) \\
&= (2y+1)(y+1)(y-1)
\end{aligned}$$