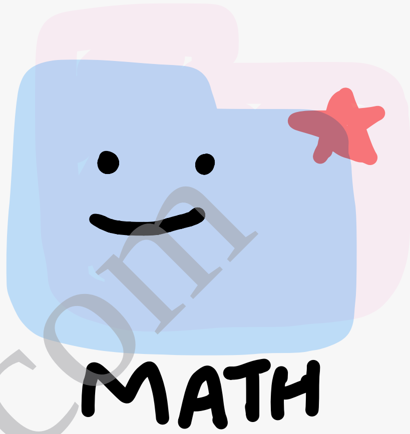
Number System
Ex. 1.5



$+ \quad \times$

$- \quad \div$

Ex. 1.5

1. (i) $2-\sqrt{5}$

$= 2 - 2.236\text{-----}$

$= 0.764\text{-----}$

Since it is a non-terminating non-repeating decimal

$\therefore$ $2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23}$

$= 3$

Since it is a terminating decimal

$\therefore$ $(3+\sqrt{23})-\sqrt{23}$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

$= \frac{2}{7}\sqrt{\frac{\cancel{7}\,1}{\cancel{7}\,1}}$ $\left[\because \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}\right]$

$= \frac{2}{7} \times 1$

$= \frac{2}{7}$, which is in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$

$\therefore$ $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

Rationalising the denominator

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

$= \frac{1.4142\text{----}}{2} = 0.7071\text{-----}$

Since it is a non-terminating non-repeating decimal.

$\therefore \frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

$= 2 \times 3.142 - - - -$

$= 6.284 - - - - -$

Since it is a non-terminating non-repeating decimal.

$\therefore 2\pi$ is an irrational number.

2 (i) $(3+\sqrt{3})(2+\sqrt{2})$

$= 3(2+\sqrt{2}) + \sqrt{3}(2+\sqrt{2})$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

$= (3)^2 - (\sqrt{3})^2$ $\quad [\because (a+b)(a-b) = a^2 - b^2]$

$= 9 - 3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

$= (\sqrt{5})^2 + (\sqrt{2})^2 + 2(\sqrt{5})(\sqrt{2})$ $\quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= 5 + 2 + 2\sqrt{10}$

$= 7 + 2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$= (\sqrt{5})^2 - (\sqrt{2})^2$ $\quad [\because (a-b)(a+b) = a^2 - b^2]$

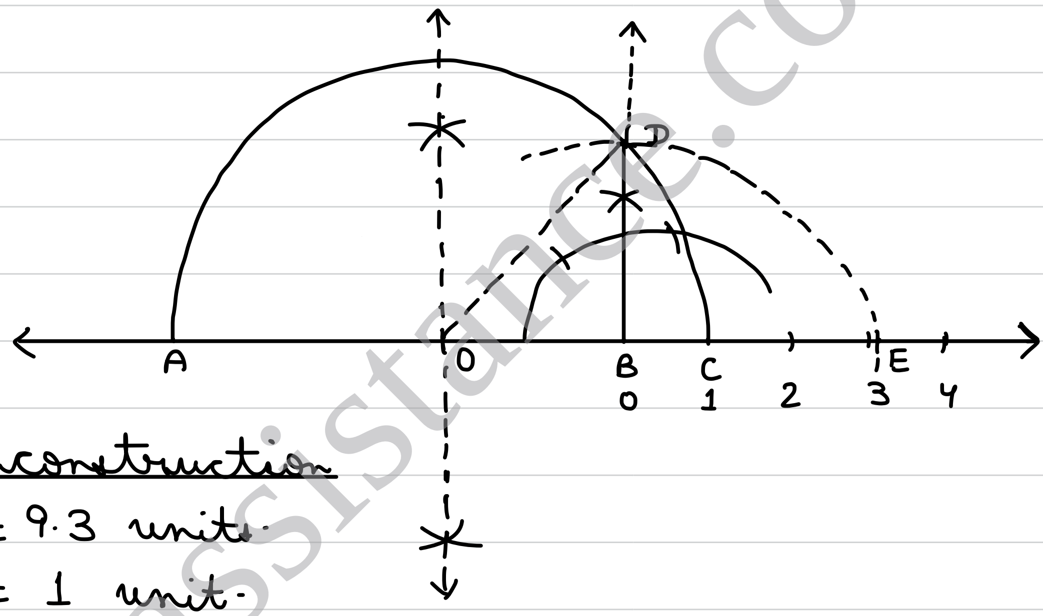
$= 5 - 2$

$= 3$

3. $\pi = \frac{c}{d}$

There is no contradiction. When we measure any length with a scale or any other device, we only get an approximate rational value. We do not realise that either 'c' or 'd' is irrational.

4. $\sqrt{9.3}$



Steps of construction

1. Draw AB = 9.3 units.
2. Draw BC = 1 unit.
3. Draw perpendicular bisector of AC intersecting AC at point O.
4. With O as centre and radius = OA or OC, draw a semicircle.
5. Draw a perpendicular at point B intersecting the semicircle at point D.
6. With B as centre and radius = BD, draw an arc intersecting the number line at point E.
7. Mark point B as 0 and C as 1.
8. With radius = 1 unit, draw arcs on the

number line representing 2, 3, 4 etc.

9. Point E represents $\sqrt{9.3}$

Justification-

Construction- Join OD.

$AB = 9.3$ units

$BC = 1$ unit

$AC = AB + BC$

$= 9.3 + 1$

$= 10.3$ units

$OA = OC = OD = \frac{AC}{2}$ (each = radius)

or $OA = OC = OD = \frac{10.3}{2}$ units —(i)

$OB = OC - BC$

$= \frac{10.3}{2} - 1$

$= \frac{10.3 - 2}{2}$

or $OB = \frac{8.3}{2}$ units —(ii)

Using Pythagoras theorem in ΔOBD

$OD^2 = OB^2 + BD^2$

or $\left(\frac{10.3}{2}\right)^2 = \left(\frac{8.3}{2}\right)^2 + BD^2$ (using eq. (i) and (ii))

or $\left(\frac{10.3}{2}\right)^2 - \left(\frac{8.3}{2}\right)^2 = BD^2$

or $\left(\frac{10.3}{2} + \frac{8.3}{2}\right)\left(\frac{10.3}{2} - \frac{8.3}{2}\right) = BD^2$ $[\because a^2 - b^2 = (a+b)(a-b)]$

or $\frac{18.6}{2} \times \frac{\cancel{2}^{1}}{\cancel{2}_{1}} = BD^2$

or $9.3 = BD^2$

Taking square root on both sides

$BD = \sqrt{9.3}$

Hence justified.

50 (i) $\frac{1}{\sqrt{7}}$

Rationalising the denominator

$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Rationalising the denominator

$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$ $[\because (a-b)(a+b) = a^2 - b^2]$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$

$= \frac{\sqrt{7}+\sqrt{6}}{1}$

$= \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Rationalising the denominator

$$\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} \quad [\because (a+b)(a-b) = a^2-b^2]$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

(v) $\frac{1}{\sqrt{7}-2}$

Rationalising the denominator

$$\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$$

$$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2} \quad [\because (a-b)(a+b) = a^2-b^2]$$

$$= \frac{\sqrt{7}+2}{7-4}$$

$$= \frac{\sqrt{7}+2}{3}$$