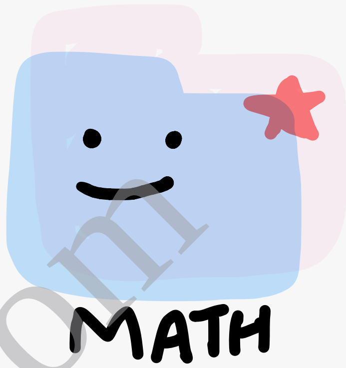
Number System
Ex. 1.3



$+ \quad \times$

$- \quad \div$

Exe. 1.3

1. (i) $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii) $\frac{1}{11} = 0.\overline{09}$, which is a non-terminating and recurring decimal.

(iii) $4\frac{1}{8} = 4.125$, which is a terminating decimal.

(iv) $\frac{3}{13} = 0.\overline{230769}$, which is a non-terminating and recurring decimal.

(v) $\frac{2}{11} = 0.\overline{18}$, which is a non-terminating and recurring decimal.

(vi) $\frac{329}{400} = 0.8225$, which is a terminating decimal

2.) $\frac{1}{7} = 0.\overline{142857}$

Yes, we can predict the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ without actually doing the long division.

$\frac{2}{7} = 2\times\frac{1}{7} = 2\times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3\times\frac{1}{7} = 3\times 0.\overline{142857} = 0.\overline{428571}$

$\frac{4}{7} = 4\times\frac{1}{7} = 4\times 0.\overline{142857} = 0.\overline{571428}$

$\frac{5}{7} = 5\times\frac{1}{7} = 5\times 0.\overline{142857} = 0.\overline{714285}$

$\frac{6}{7} = 6\times\frac{1}{7} = 6\times 0.\overline{142857} = 0.\overline{857142}$

3 (i) Let $x = 0.\overline{6}$ —— (i)

Multiplying both sides by 10

$10x = 6.\overline{6}$ — (ii)

Subtracting equation (i) from equation (ii)

$10x - x = 6.\overline{6} - 0.\overline{6}$

$9x = 6$

$x = \frac{\cancel{6}\,2}{\cancel{9}\,3}$

$x = \frac{2}{3}$

$\therefore$ $0.\overline{6} = \frac{2}{3}$, which is in the form $\frac{p}{q}$

<u>OR</u>

Let $x = 0.\overline{6}$ — (i)

Multiplying both sides by 10

$10x = 6.\overline{6}$

$10x = 6 + 0.\overline{6}$

$10x = 6 + x$ (using equation (i))

$10x - x = 6$

$9x = 6$

$x = \frac{\cancel{6}\,2}{\cancel{9}\,3}$

$x = \frac{2}{3}$

$\therefore$ $0.\overline{6} = \frac{2}{3}$, which is in the form $\frac{p}{q}$

(ii) Let $x = 0.4\overline{7}$

Multiplying both sides by 10

$10x = 4.\overline{7}$ — (i)

Multiplying both sides of equation by 10

$100x = 47.\overline{7}$ — (ii)

Subtracting equation (i) from equation (ii)

$100x - 10x = 47.\overline{7} - 4.\overline{7}$

$90x = 43$

$x = \frac{43}{90}$

$\therefore$ $0.4\overline{7} = \frac{43}{90}$, which is of the form $\frac{p}{q}$

OR

Let $x = 0.4\overline{7}$ — (i)

Multiplying both sides by 10

$10x = 4.7\overline{7}$

$10x = 4.3 + 0.4\overline{7}$

$10x = 4.3 + x$ (using equation (i))

$10x - x = 4.3$

$9x = 4.3$

$x = \frac{43}{90}$

$\therefore$ $0.4\overline{7} = \frac{43}{90}$, which is of the form $\frac{p}{q}$

(iii) Let $x = 0.\overline{001}$ — (i)

Multiplying both sides by 1000

$1000x = 1.\overline{001}$ — (ii)

Subtracting equation (i) from equation (ii)

$1000x - x = 1.\overline{001} - 0.\overline{001}$

$999x = 1$

$x = \frac{1}{999}$

$\therefore$ $0.\overline{001} = \frac{1}{999}$, which is of the form $\frac{p}{q}$

<u>OR</u>

Let $x = 0.\overline{001}$ — (i)

Multiplying both sides by 1000

$1000x = 1.\overline{001}$

$1000x = 1 + 0.\overline{001}$

$1000x = 1 + x$ (using equation (i))

$1000x - x = 1$

$999x = 1$

$x = \frac{1}{999}$

$\therefore$ $0.\overline{001} = \frac{1}{999}$, which is of the form $\frac{p}{q}$

4. Let $x = 0.\overline{9}$ — (i)

Multiplying both sides by 10

$10x = 9.\overline{9}$ — (ii)

Subtracting equation (i) from equation (ii)

$10x - x = 9.\overline{9} - 0.\overline{9}$

$9x = 9$

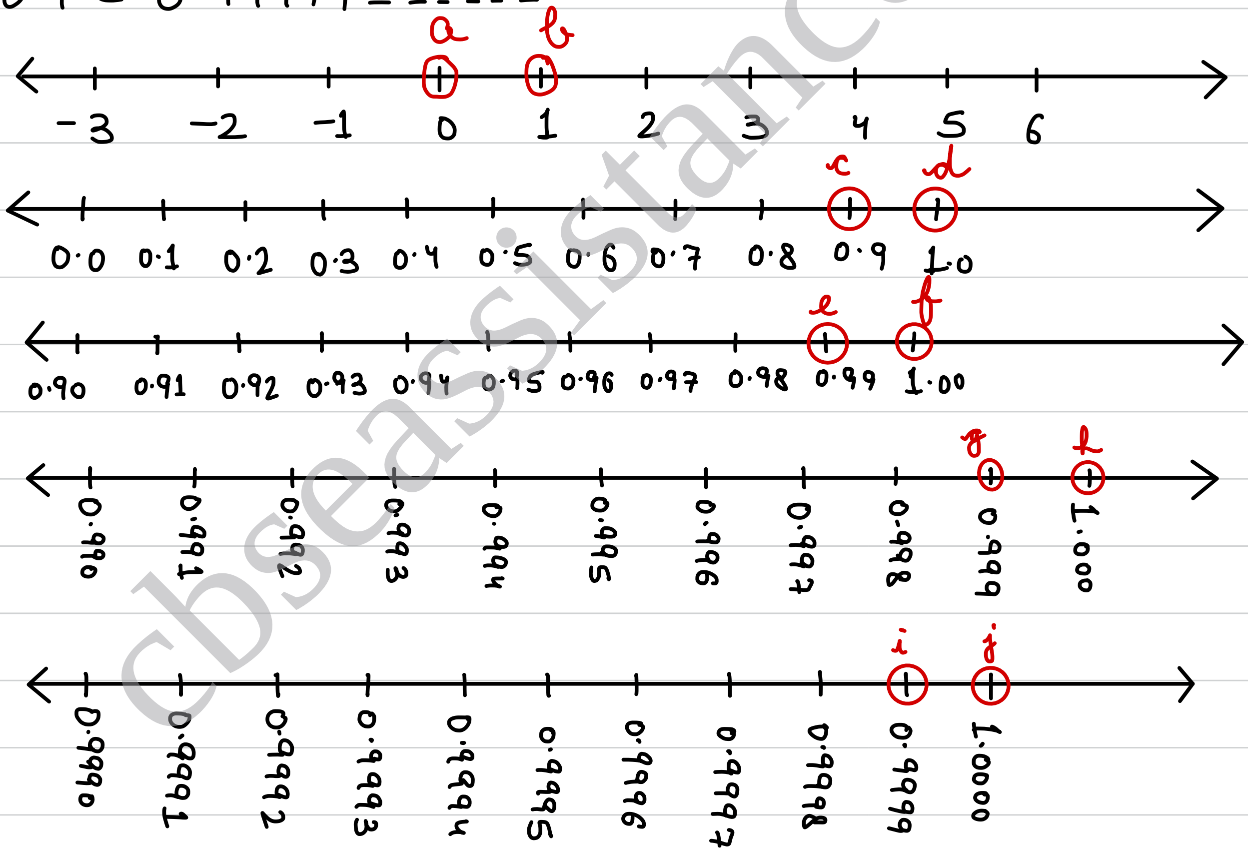
$x = \frac{\cancel{9}\,1}{\cancel{9}\,1}$

$x = 1$

Yes, as the value of $0.\overline{9} = 1$

Reason:

$0.\overline{9} = 0.99999\ldots\ldots$



We observe that as the number of digits increase in the terminating decimal, the value of $0.\overline{9}$ approaches closer to 1.

$\therefore 0.\overline{9} \approx 1$

5. The maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ is 16.

(Reason: The maximum number of digits in the repeating block of digits in the decimal expansion of a rational number is one less than divisor i.e. q)

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

$$
\begin{array}{r}
0.0588235294117647 \\
17 \enclose{longdiv}{1.00} \\
-85 \\
\hline
150 \\
-136 \\
\hline
140 \\
-136 \\
\hline
40 \\
-34 \\
\hline
60 \\
-51 \\
\hline
90 \\
-85 \\
\hline
50 \\
-34 \\
\hline
160 \\
-153 \\
\hline
70 \\
-68 \\
\hline
2
\end{array}
\qquad
\begin{array}{r}
20 \\
-17 \\
\hline
30 \\
-17 \\
\hline
130 \\
-119 \\
\hline
110 \\
-102 \\
\hline
80 \\
-68 \\
\hline
120 \\
-119 \\
\hline
1
\end{array}
$$

6. $\frac{2}{5} = 0.4$ Factors of q in each number are 5

$\frac{12}{25} = 0.48$

$25 = 5^2$

$\frac{3}{125} = 0.024$

$125 = 5^3$

$\frac{7}{10} = 0.7$

$10 = 2 \times 5$

$\frac{19}{100} = 0.19$

$100 = 2^2 \times 5^2$

$\frac{37}{1000} = 0.037$

$1000 = 2^3 \times 5^3$

$\frac{3}{2} = 1.5$

2

$\frac{3}{4} = 0.75$

$4 = 2^2$

$\frac{13}{8} = 1.625$

$8 = 2^3$

The factors of q are 2 or 5 or both 2 and 5.

7. Three numbers whose decimal expansions are non-terminating non-recurring are

0.5755755575555 7 - - - - - - -

23.14114111411114 - - - - - -

7.29229222922229 - - - - - -

8. $\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.8181$ - - - - -

Three irrational numbers are

0.7215115111511115 - - - - -

0.7632332333233332 - - - - -

0.7948448444844448 - - - - - -

9 (i) $\sqrt{23} = 4.79583152331272$ - - -

Since the decimal is non-terminating non-repeating

$\therefore \sqrt{23}$ is an irrational number.

(ii) $\sqrt{225} = \sqrt{\underline{3\times3}\times\underline{5\times5}} = 3\times5 = 15$, which is rational

$\therefore \sqrt{225}$ is a rational number.

(iii) 0.3796

Since the decimal is terminating.

$\therefore$ 0.3796 is a rational number.

(iv) 7.478478 - - - - -

Since the decimal is non-terminating repeating.

$\therefore$ 7.478478 - - - - is a rational number.

(v) 1.101001000100001 - - - - -

Since the decimal is non-terminating non-repeating.

$\therefore$ 1.101001000100001 - - - is an irrational number.