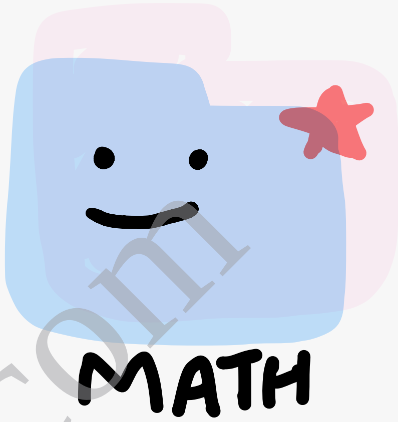
Real Numbers
Ex. 1.3



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Ex. 1.3

1. Let us suppose that $\sqrt{5}$ is a rational number.

$\therefore$ $\sqrt{5} = \frac{a}{b}$, where 'a' and 'b' are integers and $b \neq 0$.

Dividing 'a' and 'b' by their HCF

$\sqrt{5} = \frac{p}{q}$, where 'p' and 'q' are integers, $q \neq 0$ and p, q are co-prime.

or $p = \sqrt{5}\, q$

Squaring both sides

$p^2 = 5q^2$ — (i)

$\therefore$ p^2 is divisible by 5

or p is also divisible by 5 — (ii)

$\therefore$ $p = 5m$, where 'm' is an integer

Squaring both sides

$p^2 = 25m^2$

or $5q^2 = 25m^2$ (using equation (i))

or $q^2 = \frac{\cancel{25}^{5} m^2}{\cancel{5}_{1}}$

or $q^2 = 5m^2$

$\therefore$ q^2 is divisible by 5.

or q is also divisible by 5 — (iii)

From (ii) and (iii)

p and q have 5 as a common factor.

But this contradicts the fact that p and q are co-prime.

$\therefore$ Our supposition that $\sqrt{5}$ is rational is wrong.

$\sqrt{5}$ is not a rational number.

$\therefore$ $\sqrt{5}$ is an irrational number.

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2. Let us suppose that $3+2\sqrt{5}$ is a rational number.

$\therefore 3+2\sqrt{5} = \frac{a}{b}$, where 'a' and 'b' are integers and $b \neq 0$.

Dividing a and b by their HCF

$3+2\sqrt{5} = \frac{p}{q}$, where 'p' and 'q' are integers, $q \neq 0$, p and q are co-prime.

or $2\sqrt{5} = \frac{p}{q} - 3$

or $2\sqrt{5} = \frac{p-3q}{q}$

or $\sqrt{5} = \frac{p-3q}{2q}$

2, 3, p and q are integers

$\therefore \frac{p-3q}{2q}$ is a rational number.

So $\sqrt{5}$ is also a rational number.

But this contradicts the fact that $\sqrt{5}$ is an irrational number.

Our supposition that $\sqrt{5}$ is rational is wrong

$\therefore \sqrt{5}$ is not a rational number.

$\therefore \sqrt{5}$ is an irrational number

3(i) Let us suppose that $\frac{1}{\sqrt{2}}$ is a rational number.

$\therefore \frac{1}{\sqrt{2}} = \frac{a}{b}$, where 'a' and 'b' are integers and $b \neq 0$

Dividing a and b by their H.C.F.

$\frac{1}{\sqrt{2}} = \frac{p}{q}$, where 'p' and 'q' are integers, $q \neq 0$, p and q are co-prime.

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or $\sqrt{2} = \frac{q}{p}$

p and q are integers

$\therefore \frac{q}{p}$ is a rational number

So $\sqrt{2}$ is also a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

$\therefore$ Our supposition is wrong.

$\frac{1}{\sqrt{2}}$ is not a rational number.

$\therefore \frac{1}{\sqrt{2}}$ is an irrational number.

(11) Let us suppose that $7\sqrt{5}$ is a rational number

$\therefore 7\sqrt{5} = \frac{a}{b}$, where 'a' and 'b' are integers and $b \neq 0$

Dividing a and b by their H.C.F.

$7\sqrt{5} = \frac{p}{q}$, where 'p' and 'q' are integers, $q \neq 0$, p and q are co-prime.

or $\sqrt{5} = \frac{p}{7q}$

$p, q, 7$ are integers

$\therefore \frac{p}{7q}$ is a rational number

So, $\sqrt{5}$ is also a rational number.

But this contradicts the fact that $\sqrt{5}$ is an irrational number.

$\therefore$ Our supposition is wrong.

$7\sqrt{5}$ is not a rational number.

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$\therefore$ $7\sqrt{5}$ is an irrational number.

(11) Let us suppose that $6+\sqrt{2}$ is a rational number.

$\therefore$ $6+\sqrt{2} = \frac{a}{b}$, where 'a' and 'b' are integers, $b \neq 0$

Dividing a and b by their H.C.F.

$6+\sqrt{2} = \frac{p}{q}$, where 'p' and 'q' are integers, $q \neq 0$, p and q are co-prime.

or $\sqrt{2} = \frac{p}{q} - 6$

or $\sqrt{2} = \frac{p-6q}{q}$

p, q, 6 are integers

$\therefore$ $\frac{p-6q}{q}$ is a rational number.

So, $\sqrt{2}$ is also a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

$\therefore$ Our supposition is wrong.

$\therefore$ $6+\sqrt{2}$ is not a rational number.

$\therefore$ $6+\sqrt{2}$ is an irrational number.

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