Real Numbers
Ex. 1.1

$+ \quad \times$

$- \quad \div$

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1 (i) Using Euclid's division lemma

$225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$

Using Euclid's division lemma

$135 = 90 \times 1 + 45$

Since remainder $45 \neq 0$

Using Euclid's division lemma

$90 = 45 \times 2 + 0$

Since remainder $= 0$

$\therefore$ H.C.F. $= 45$

(ii) Using Euclid's division lemma

$38220 = 196 \times 195 + 0$

Since remainder $= 0$

$\therefore$ H.C.F. $= 196$

(iii) Using Euclid's division lemma

$867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$

Using Euclid's division lemma

$255 = 102 \times 2 + 51$

Since remainder $51 \neq 0$

Using Euclid's division lemma

$102 = 51 \times 2 + 0$

Since remainder $= 0$

$\therefore$ H.C.F. $= 51$

2. Let 'a' be any positive integer and $b = 6$.
By Euclid's algorithm, $a = 6q + r$, where $0 \leq r < 6$

$\therefore$ Possible values of 'r' are 0, 1, 2, 3, 4, 5

When $r = 0$, $a = 6q + 0$

or $a = 6q = 2(3q) = 2m$, where $m = 3q$

$\therefore$ $a = 6q$ is even

When $r = 1$, $a = 6q + 1$

$= 2(3q) + 1$

$= 2m + 1$, where $m = 3q$

$\therefore$ $a = 6q + 1$ is odd

When $r = 2$, $a = 6q + 2$

$= 2(3q + 1)$

$= 2m$, where $m = 3q + 1$

$\therefore$ $a = 6q + 2$ is even

When $r = 3$, $a = 6q + 3$

$= 6q + 2 + 1$

$= 2(3q + 1) + 1$

$= 2m + 1$, where $m = 3q + 1$

$\therefore$ $a = 6q + 3$ is odd

When $r = 4$, $a = 6q + 4$

$= 2(3q + 2)$

$= 2m$, where $m = 3q + 2$

$\therefore$ $a = 6q + 4$ is even

When $r = 5$, $a = 6q + 5$

$= 6q + 4 + 1$

$= 2(3q + 2) + 1$

$= 2m + 1$, where $m = 3q + 2$

$\therefore$ $a = 6q + 5$ is odd

∴ Any positive odd integer is of the form $6q+1$ or $6q+3$ or $6q+5$, where 'q' is some integer.

3. Number of members in army contingent $= 616$
Number of members in army band $= 32$
The maximum number of columns in which they can march is the H.C.F. of 616 and 32.
Using Euclid's division lemma
$616 = 32 \times 19 + 8$
Since remainder $8 \neq 0$
Using Euclid's division lemma
$32 = 8 \times 4 + 0$
Since remainder $= 0$
∴ H.C.F. $= 8$
∴ The maximum number of columns in which they can march is 8.

4. Let 'a' be any positive integer and $b = 3$.
∴ By Euclid's division lemma, $a = 3q + r,\ 0 \leq r < 3$
Possible values of 'r' are $0, 1, 2$
When $r = 0$, $a = 3q + 0$
$a = 3q$
Squaring both sides
$a^2 = 9q^2$
$= 3(3q^2)$
$= 3m$, where $m = 3q$
When $r = 1$, $a = 3q + 1$
Squaring both sides

$a^2 = (3q+1)^2$

$= (3q)^2 + 2(3q)(1) + (1)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$, where $m = 3q^2 + 2q$

When $r = 2$, $a = 3q + 2$

Squaring both sides

$a^2 = (3q+2)^2$

$= (3q)^2 + 2(3q)(2) + (2)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$, where $m = 3q^2 + 4q + 1$

$\therefore$ The square of any positive integer is either of the form $3m$ or $3m+1$, for some integer m.

5. Let 'a' be any positive integer and $b = 3$

$\therefore$ By Euclid's division lemma, $a = 3q + r$, $0 \le r < 3$

Possible values of 'r' are 0, 1, 2

When $r = 0$, $a = 3q + 0$

$a = 3q$

Cubing both sides

$a^3 = 27q^3$

$= 9(3q^3)$

$= 9m$, where $m = 3q^3$

When $r = 1$, $a = 3q + 1$

Cubing both sides

$a^3 = (3q+1)^3$

$= (3q)^3 + 3(3q)(1)(3q+1) + (1)^3$ $[(a+b)^3 = a^3 + 3ab(a+b) + b^3]$

$= 27q^3 + 9q(3q+1) + 1$

$= 3[9q^3 + 3q(3q+1)] + 1$

$= 3m+1$, where $m = 9q^3 + 3q(3q+1)$

When $r=2$, $a = 3q+2$

Cubing both sides

$a^3 = (3q+2)^3$

$= (3q)^3 + 3(3q)(2)(3q+2) + (2)^3$

$= 27q^3 + 18q(3q+2) + 8$

$= 3[9q^3 + 6q(3q+2)] + 8$

$= 3m+8$, where $m = 9q^3 + 6q(3q+2)$

$\therefore$ The cube of any positive integer is of the form $9m$, $9m+1$ or $9m+8$.