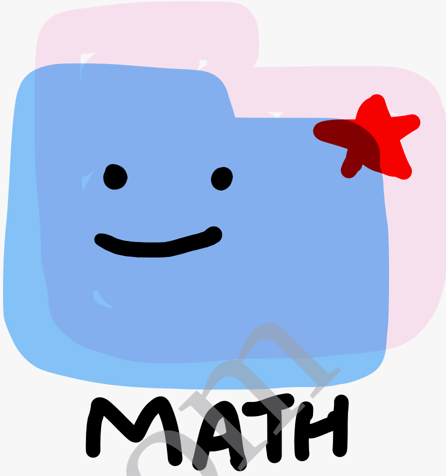
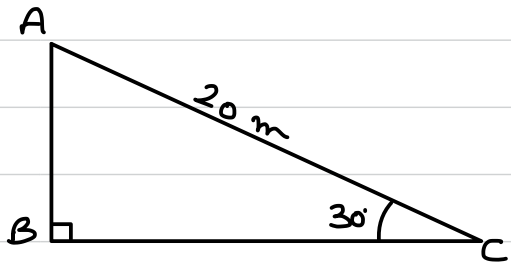
Some Applications Of Trigonometry



$+ \quad \times$

$- \quad \div$

Ex. 9.1



1. Length of rope, AC = 20 m

$\angle ACB = 30^\circ$

Let AB = h metres represent the pole

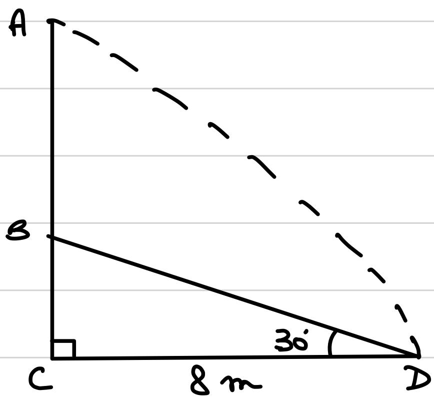
In $\triangle ABC$, $\sin 30^\circ = \frac{AB}{AC}$

or $\frac{1}{2} = \frac{AB}{20}$

or $AB = \frac{\cancel{20}^{10}}{\cancel{2}_{1}}$

or AB = 10 m

$\therefore$ Height of pole = 10 m



2. Let AC represent the tree which breaks at point B and the top touches at point D and AB = BD.

CD = 8 m, $\angle BDC = 30^\circ$

In $\triangle BCD$, $\tan 30^\circ = \frac{BC}{CD}$

or $\frac{1}{\sqrt{3}} = \frac{BC}{8}$

or $BC = \frac{8}{\sqrt{3}}$

or $BC = \frac{8}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

or $BC = \frac{8\sqrt{3}}{3}$ m

In $\triangle BCD$, $\cos 30^\circ = \frac{CD}{BD}$

or $\frac{\sqrt{3}}{2} = \frac{8}{BD}$

or $BD = \frac{16}{\sqrt{3}}$

or $BD = \frac{16}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

or $BD = \frac{16\sqrt{3}}{3}$ m

$AB = BD = \frac{16\sqrt{3}}{3}$ m

$\therefore\ AC = AB + BC$

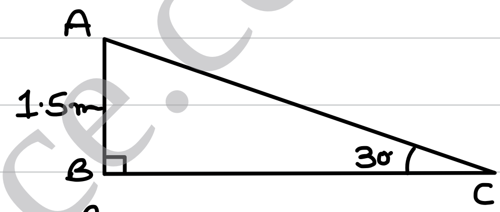
$= \frac{16\sqrt{3}}{3} + \frac{8\sqrt{3}}{3}$

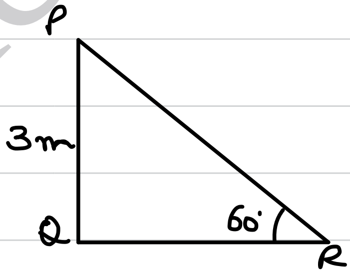
$= \frac{\cancel{24}^{\,8}\sqrt{3}}{\cancel{3}_{\,1}}$

$= 8\sqrt{3}$ m

$\therefore$ Height of tree $= 8\sqrt{3}$ m

3. Let AC and PR represent the two slides with $AB = 1.5$ m, $PQ = 3$ m, $\angle ACB = 30°$, $\angle PRQ = 60°$





In $\triangle ABC$, $\sin 30° = \frac{AB}{AC}$

or $\frac{1}{2} = \frac{1.5}{AC}$

or $AC = 1.5 \times 2$

or $AC = 3$ m

In $\triangle PQR$, $\sin 60° = \frac{PQ}{PR}$

or $\frac{\sqrt{3}}{2} = \frac{3}{PR}$

or $PR = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

or $PR = \frac{\cancel{6}^{\,2}\sqrt{3}}{\cancel{3}_{\,1}}$

or $PR = 2\sqrt{3}$ m

$\therefore$ Lengths of slides are 3 m and $2\sqrt{3}$ m.

4. $BC = 30\ m$

$\angle ACB = 30°$

Let AB represent the tower of height 'h' metres

In ΔABC, $\tan 30° = \frac{AB}{BC}$

or $\frac{1}{\sqrt{3}} = \frac{AB}{BC}$

or $\frac{1}{\sqrt{3}} = \frac{AB}{30}$

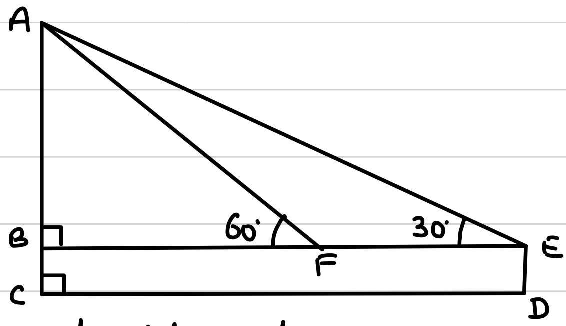
or $AB = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

or $AB = \frac{\cancel{30}^{10}\sqrt{3}}{\cancel{3}_{1}}$

or $AB = 10\sqrt{3}$

$\therefore$ Height of tower $= 10\sqrt{3}\ m$

5. Let $AC = 30\ m$ represent the building and $DE = 1.5\ m$ represent the boy.



$\angle AEB = 30°$, $\angle AFB = 60°$

Let the boy walks from point E to F.

Since BCDE is a rectangle and opposite sides of a rectangle are equal.

$\therefore\ BC = DE = 1.5m$

$AB = AC - BC$

$= 30 - 1.5$

$= 28.5\ m$

Let $BF = x$ metres and $EF = y$ metres

$\therefore BE = BF + EF$

$= (x+y)$ metres

In $\triangle ABF$, $\tan 60^\circ = \frac{AB}{BF}$

or $\sqrt{3} = \frac{28.5}{x}$

or $x = \frac{28.5}{\sqrt{3}}$ —— (i)

In $\triangle ABE$, $\tan 30^\circ = \frac{AB}{BE}$

or $\frac{1}{\sqrt{3}} = \frac{28.5}{x+y}$

or $x+y = 28.5\sqrt{3}$

or $\frac{28.5}{\sqrt{3}} + y = 28.5\sqrt{3}$

or $y = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$

or $y = 28.5\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

or $y = 28.5\left(\frac{3-1}{\sqrt{3}}\right)$

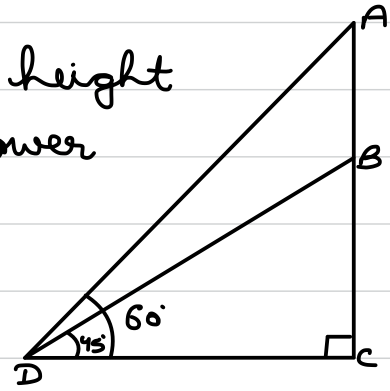
or $y = \frac{28.5 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

or $y = \frac{\overset{9.5}{\cancel{28.5}} \times 2 \times \sqrt{3}}{\cancel{3}}$

or $y = 19\sqrt{3}$

$\therefore$ Distance walked towards the building $= 19\sqrt{3}$ m

7. Let BC represent the building of height 20 m and AB represent the tower of height 'h' metres



$\angle ADC = 60°$, $\angle BDC = 45°$

In ΔBCD, $\tan 45° = \frac{BC}{CD}$

or $1 = \frac{20}{CD}$

or $CD = 20$ m

$AC = AB + BC$

or $AC = (h+20)$ metres

In ΔACD, $\tan 60° = \frac{AC}{CD}$

or $\sqrt{3} = \frac{h+20}{20}$

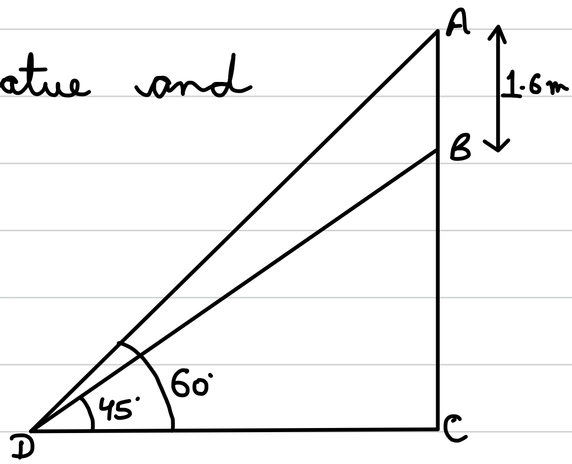
or $20\sqrt{3} = h + 20$

or $20\sqrt{3} - 20 = h$

or $h = 20(\sqrt{3}-1)$

$\therefore$ Height of tower $= 20(\sqrt{3}-1)$ m

8. Let AB = 1.6 m represent the statue and BC represents the pedestal



$\angle BDC = 45°$

$\angle ADC = 60°$

Let BC = h metre

CD = x metre

In ΔBCD, $\tan 45° = \frac{BC}{CD}$

$1 = \frac{h}{x}$ or $h = x$ —— (1)

In ΔACD, $\tan 60^\circ = \frac{AC}{CD}$

or $\sqrt{3} = \frac{AB+BC}{x}$

or $\sqrt{3} = \frac{1.6+h}{x}$

or $\sqrt{3}x = 1.6+h$

or $\sqrt{3}h = 1.6 + h$ (using eq. (1))

or $\sqrt{3}h - h = 1.6$

or $h(\sqrt{3}-1) = 1.6$

or $h = \frac{1.6}{\sqrt{3}-1}$

Rationalising the denominator

$$h = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$h = \frac{1.6(\sqrt{3}+1)}{(\sqrt{3})^2-(1)^2} \qquad [\because (a+b)(a-b) = a^2-b^2]$$

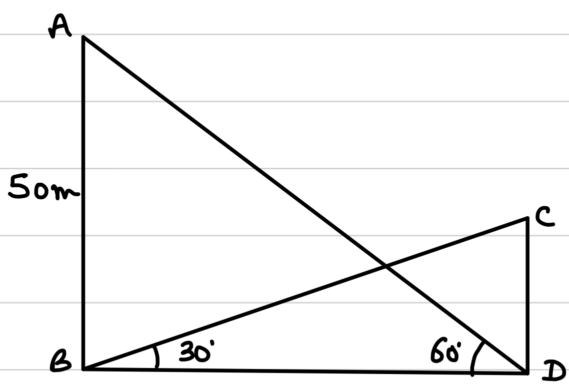
$$h = \frac{1.6(\sqrt{3}+1)}{3-1}$$

$$h = \frac{\cancel{1.6}^{0.8}(\sqrt{3}+1)}{\cancel{2}\,1}$$

$$h = 0.8(\sqrt{3}+1)$$

$\therefore$ Height of pedestal $= 0.8(\sqrt{3}+1)$ m

9. Let AB represents the tower and CD represents the building.



$AB = 50$ m

$\angle ADB = 60^\circ$

$\angle CDB = 30°$

Let $CD = h$ metres

and $BD = x$ metres

In ΔABD, $\tan 60° = \frac{AB}{BD}$

or $\sqrt{3} = \frac{50}{x}$

or $x = \frac{50}{\sqrt{3}}$ m ——— (1)

In ΔCBD, $\tan 30° = \frac{CD}{BD}$

or $\frac{1}{\sqrt{3}} = \frac{h}{x}$

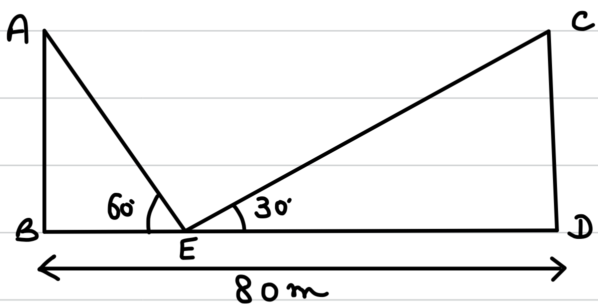
or $\frac{x}{\sqrt{3}} = h$

or $\frac{50}{\sqrt{3} \times \sqrt{3}} = h$ (using eq^n: (1))

or $\frac{50}{3} = h$

or $h = 16\frac{2}{3}$ m

$\therefore$ Height of building $= 16\frac{2}{3}$ m



10. Let AB and CD be the two poles of equal height.

$\angle AEB = 60°$

$\angle CED = 30°$

$BD = 80$ m

Let $AB = CD = h$ metres

Let $BE = x$ metres

$\therefore DE = BD - BE$

$DE = (80 - x)$ metres

In ΔABE

$\tan 60^\circ = \frac{AB}{BE}$

or $\sqrt{3} = \frac{h}{x}$

or $h = \sqrt{3}x$ —— ①

In ΔCDE

$\tan 30^\circ = \frac{CD}{DE}$

or $\frac{1}{\sqrt{3}} = \frac{h}{80 - x}$

or $80 - x = \sqrt{3}h$

or $80 - x = \sqrt{3} \times \sqrt{3}x$ (using eq^n. ①)

or $80 - x = 3x$

or $80 = 3x + x$

or $80 = 4x$

or $x = \frac{\cancel{80}^{20}}{\cancel{4}_{1}}$

or $x = 20$

Put the value of x in eq^n. ①

$h = 20\sqrt{3}$ m

$\therefore BE = 20$ m

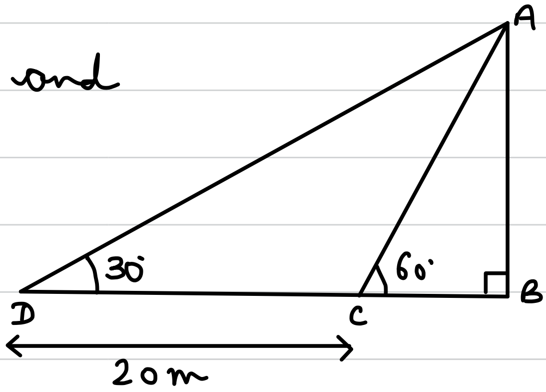
$DE = 80 - 20$

$= 60$ m

$\therefore$ Height of building $= 20\sqrt{3}$ m

Distance of point E from the two poles is 20 m and 60 m

11. Let AB represents the tower and BC represents the canal.



$\angle ACB = 60°$

$\angle ADB = 30°$

$CD = 20\,m$

Let $AB = h$ metres

$BC = x$ metres

$BD = BC + CD$

or $BD = (x+20)\,m$

In ΔABC

$\tan 60° = \frac{AB}{BC}$

or $\sqrt{3} = \frac{h}{x}$ $\quad (\because \tan 60° = \sqrt{3})$

or $h = \sqrt{3}\,x$ ——— ①

In ΔABD

$\tan 30° = \frac{AB}{BD}$ $\quad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$

or $\frac{1}{\sqrt{3}} = \frac{h}{x+20}$

or $x+20 = \sqrt{3}\,h$

or $x+20 = \sqrt{3} \times \sqrt{3}\,x$ (using eqⁿ ①)

or $x+20 = 3x$

or $20 = 3x - x$

or $20 = 2x$

or $x = \frac{20}{2}$

or $x = 10$

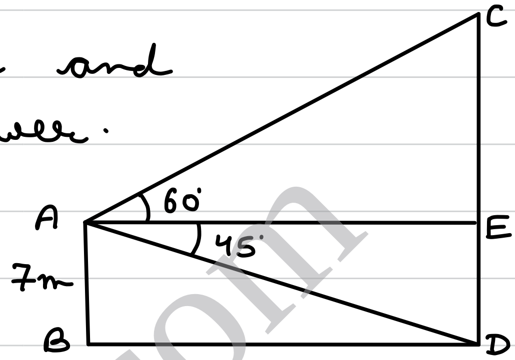
Put the value of x in equation ①

$h = 10\sqrt{3}$

$\therefore$ Height of the tower $= 10\sqrt{3}$ m

Width of the river $= 10$ m

12. Let AB represents the building and CD represents the cable tower.



$AB = 7$ m

$\angle CAE = 60°$

$\angle DAE = 45°$

Let $BD = x$ metres

$CE = h$ metres

Since each angle of quadrilateral $ABDE = 90°$

$\therefore$ ABDE is a rectangle.

Opposite sides of a rectangle are equal.

$\therefore BD = AE = x$ metres

$AB = DE = 7$ metres

$\therefore CD = CE + DE$

or $CD = (h+7)$ metres

In $\triangle ADE$, $\tan 45° = \dfrac{DE}{AE}$

or $1 = \dfrac{7}{x}$ $\quad (\because \tan 45° = 1)$

or $x = 7$ ——— (1)

In $\triangle CEA$, $\tan 60° = \dfrac{CE}{AE}$

or $\sqrt{3} = \dfrac{h}{x}$ $\quad (\because \tan 60° = \sqrt{3}x)$

or $h = \sqrt{3}x$

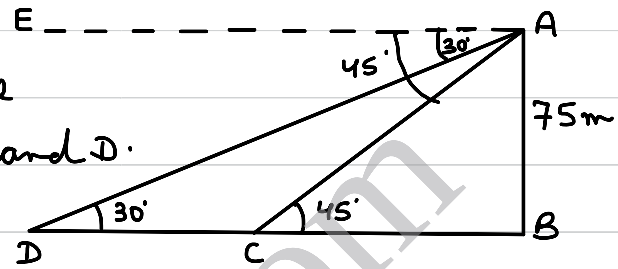
or $h = 7\sqrt{3}$ (using eq^n. (1))

$\therefore$ Height of tower $= CD$

$= h + 7$

$= 7\sqrt{3} + 7$

Height of tower $= 7(\sqrt{3}+1)$ m



13. Let AB represents the lighthouse

The two ships are at points C and D.

$AB = 75$m

$\angle EAD = \angle ACB = 45°$ (alternate

$\angle EAC = \angle ADB = 30°$ interior angles as $AE \parallel BD$)

Let $BC = x$ metres

$CD = y$ metres

In $\triangle ABC$

$\tan 45° = \frac{AB}{BC}$

or $1 = \frac{75}{x}$ $(\because \tan 45° = 1)$

or $x = 75$ ——— ①

In $\triangle ABD$

$\tan 30° = \frac{AB}{BD}$ $(\because \tan 30° = \frac{1}{\sqrt{3}})$

or $\frac{1}{\sqrt{3}} = \frac{75}{x+y}$

or $x + y = 75\sqrt{3}$

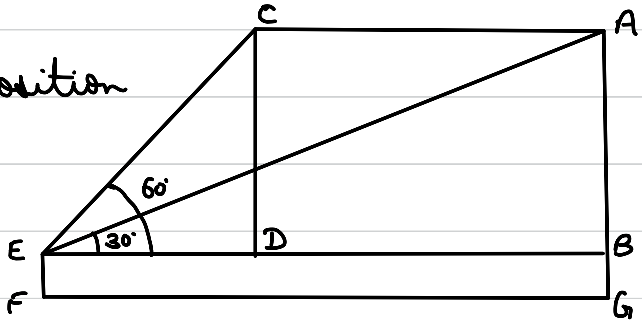
or $75 + y = 75\sqrt{3}$ (using equation ①)

or $y = 75\sqrt{3} - 75$

or $y = 75(\sqrt{3} - 1)$

$\therefore$ The distance between the two ships $= 75(\sqrt{3}-1)$ m

14. Let A and C represent the position of the balloon.



Let EF represents the girl

$EF = 1.2\ m$

$AG = 88.2\ m$

The distance travelled by the balloon is AC.

Since each angle of quadrilateral BGFE and ABDC $= 90°$

$\therefore$ BGFE and ABDC are rectangles.

Opposite sides of a rectangle are equal.

$\therefore\ EF = BG = 1.2\ m$

$AB = AG - BG$

or $AB = 88.2 - 1.2$

or $AB = 87\ m$

$\therefore\ AB = CD = 87\ m$

Let $BD = x$ metres

$\therefore\ BD = AC = x$ metres

Let $DE = y$ metres

In ΔCDE, $\tan 60° = \frac{CD}{DE}$

or $\sqrt{3} = \frac{87}{y}$ $\quad (\because \tan 60° = \sqrt{3})$

or $y = \frac{87}{\sqrt{3}}$ —— (1)

In ΔABE

$\tan 30° = \frac{AB}{BE}$

or $\frac{1}{\sqrt{3}} = \frac{87}{x+y}$ $\quad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$

or $x + y = 87\sqrt{3}$

or $x + \frac{87}{\sqrt{3}} = 87\sqrt{3}$ (using equation ①)

or $x = 87\sqrt{3} - \frac{87}{\sqrt{3}}$

or $x = 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

or $x = 87\left(\frac{3-1}{\sqrt{3}}\right)$

or $x = \frac{87 \times 2}{\sqrt{3}}$

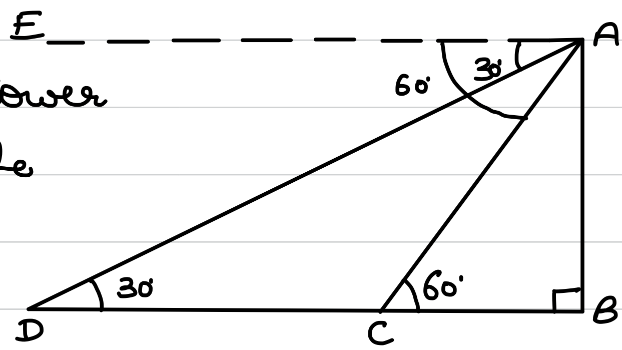
Rationalising the denominator

$x = \frac{87 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

or $x = \frac{\overset{29}{\cancel{87}} \times 2\sqrt{3}}{\underset{1}{\cancel{3}}}$

or $x = 58\sqrt{3}$

$\therefore$ Distance travelled by balloon $= 58\sqrt{3}$ m



15. Let AB represents the tower and points C and D the position of the car.

$\angle EAD = 30°$

$\angle EAC = 60°$

$\angle BDA = \angle EAD = 30°$ (alternate interior angles as AE ∥ BD)

$\angle BCA = \angle EAC = 60°$

Let BC = x metres

CD = y metres

AB = h metres

$BD = BC + CD$

or $BD = (x+y)$ metres

Let the speed of car be 'v' m/s

In $\triangle ABC$

$\tan 60° = \frac{AB}{BC}$

or $\sqrt{3} = \frac{h}{x}$ $\quad (\because \tan 60° = \sqrt{3})$

or $h = \sqrt{3}x$ —— (i)

In $\triangle ABD$

$\tan 30° = \frac{AB}{BD}$

or $\frac{1}{\sqrt{3}} = \frac{h}{x+y}$ $\quad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$

or $x+y = \sqrt{3}h$

or $x+y = \sqrt{3} \times \sqrt{3}x$ (using equation (i))

or $x+y = 3x$

or $y = 3x - x$

or $y = 2x$

or $x = \frac{y}{2}$ —— (ii)

Distance travelled by car in 6 seconds, y = speed × time

or $y = v \times 6$

or $y = 6v$

Let the time taken by car to travel from point C to B be 't' seconds

∴ Distance travelled by car in 't' seconds,

$x = vt$

Put the values of x and y in equation (ii)

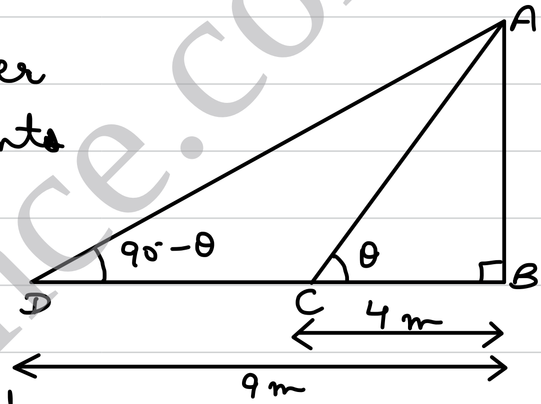
$v\,t = \frac{6v}{2}$

or $t = \frac{\cancel{6}^{3}\cancel{v}}{\cancel{2}_{1}\cancel{v}}$

or $t = 3$ seconds

∴ Time taken by the car to reach the foot of the tower from point C = 3 seconds

16. Let AB represents the tower and C and D be the points of observation.



Let AB = h metres

Let ∠ACB = θ

∴ ∠ADB = 90° − θ (as ∠ACB and ∠ADB are complementary)

In △ACB

$\tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{h}{4}$ —— (i)

In △ABD

$\tan(90° - \theta) = \frac{AB}{BD}$

or $\cot\theta = \frac{h}{9}$ —— (ii)

Multiplying equations (i) and (ii)

$\tan\theta \times \cot\theta = \frac{h}{4} \times \frac{h}{9}$

or $\frac{\cancel{\tan\theta}^{1}}{} \times \frac{1}{\cancel{\tan\theta}_{1}} = \frac{h^2}{36}$

or $1 = \frac{h^2}{36}$

or $h^2 = 36$

Taking square root on both sides

$h = \pm 6$

We reject (-6) as height cannot be negative.

$\therefore h = 6$ m

$\therefore$ Height of the tower $= 6$ m