Basic Concepts

1. An algebraic expression in which the variables involved have only non – negative integral powers is called a polynomial. For example, $x^2 + 5x - 6, x^3 - 7x^2 + 11, x^5 - 3x + 2, x^2 + \sqrt{5}, x^4 + 5x^3 - 2x^2 + 7x - 3$, etc. are polynomials.
2. In the $5x^3 - 4x^2 + 6x - 3$, we say that the *coefficients* of x^3, x^2 and x are $5, -4$ and 6 respectively, and we also say that (-3) is the constant term in it.
3. In case of a polynomial in one variable, the highest power of the variable is called the *degree of the polynomial*. For example, $2x + 3$ is a polynomial in x of degree $1, 4x^2 - \frac{3}{2}x - 5$ is a polynomial in x of degree 2, and $3x^4 - 5x^2 + 1$ is a polynomial in x of degree 4.
4. A polynomial of degree 1 is called a *linear polynomial*. For example, $3x + 5$ is a linear polynomial in x.
5. A polynomial of degree 2 is called a *quadratic polynomial*. For example, $x^2 + 5x - \frac{1}{2}$ is a quadratic polynomial in x.
6. A polynomial of degree 3 is called a *cubic polynomial*. For example, $4x^3 - 3x^2 + 7x + 1$ is a cubic polynomial in x.
7. A polynomial of degree 4 is called a *biquadratic polynomial*. For example, $x^4 - 3x^3 + 2x^2 + 5x - 3$ is a biquadratic polynomial in x.
8. A polynomial having one term is called a *monomial*. Thus, $5x, 7x^2, 11x^3, 3xy$ and $2xyz$ are some examples of monomials in one, two and three variables.
9. A polynomial having two terms is called a *binomial*. Thus, $x + 1, 2x^3 + 5, x^2 - 1, x^6 + 1, x + y, x^2 + y^2$ are some examples of binomials in one and two variables.
10. A polynomial having three terms is called a *trinomial*. Thus, $x^2 - 3x + 1, x^3 - 7x^2 + 11, x + y + z$ are some examples of trinomials.

11. A polynomial containing one term only, consisting of a constant is called a *constant polynomial*. For example, $3, -5, \frac{7}{8}$, etc. are all constant polynomials. In general, every real number is a constant polynomial. The degree of a non – zero constant polynomial is zero.
12. A polynomial consisting of one term namely zero only, is called a zero polynomial. The degree of zero polynomial is not defined.
13. Let $p(x)$ be a polynomial. If $p(\alpha) = 0$, then we say that α is a zero of the polynomial $p(x)$.
    Finding the zeroes of a polynomial $p(x)$ means solving the equation $p(x) = 0$.
14. The constant polynomial has no zero.
15. Every real number is a zero of the zero polynomial.
16. A linear polynomial has one and only one zero.
17. If a polynomial $p(x)$ is divided by $g(x) = x - a$, then the remainder is given by $p(a)$ where degree of $p(x) \geq$ degree of $g(x)$
18. **Factor Theorem**: Let $p(x)$ be a polynomial of degree $n > 1$ and let a be any real number.

(i) If $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.
(ii) If $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

19. Following identities are true for all values of the variables a, b and c.

(i) $(a + b)^2 = a^2 + 2ab + b^2$
(ii) $(a - b)^2 = a^2 - 2ab + b^2$
(iii) $(a + b)(a - b) = a^2 - b^2$
(iv) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
(v) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
(vi) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
(vii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
(viii) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
(ix) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$