

Basic Concepts

1. A number is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
2. There are infinitely many rational numbers between any two given rational numbers.
3. A number is called an irrational number if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
4. All the rational and irrational numbers make up the collection of real numbers.
5. Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.
6. The decimal expansion of a rational number is either terminating or non – terminating repeating. Moreover, a number whose decimal expansion is terminating or non – terminating repeating, is rational.
7. The decimal expansion of an irrational number is non – terminating non – repeating. Moreover, a number whose decimal expansion is non – terminating non – repeating, is irrational.
8. The sum or difference of a rational number and an irrational number is irrational.
9. The product or quotient of a non – zero rational number with an irrational number is irrational.
10. If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.
11. For positive real numbers a and b , the following identities hold:
 - (i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$
 - (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
 - (iii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

$$(iv) \quad (a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

$$(v) \quad (\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$$

12. When the denominator of an expression contains a term with a square root (or a number under a radical sign), the process of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator.

13. To rationalise the denominator of $\frac{1}{\sqrt{a+b}}$, we multiply this by $\frac{\sqrt{a-b}}{\sqrt{a-b}}$, where a and b are integers.

14. Let $a > 0$ be a real number and p and q be rational numbers, then

$$(i) \quad a^p \cdot a^q = a^{p+q}$$

$$(ii) \quad (a^p)^q = a^{pq}$$

$$(iii) \quad \frac{a^p}{a^q} = a^{p-q}, p > q$$

$$(iv) \quad a^p \cdot b^p = (ab)^p$$