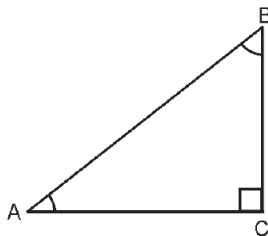
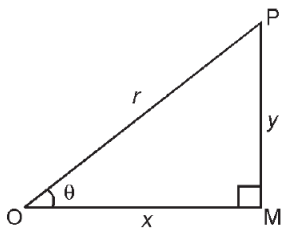


### Basic Concepts

1. In trigonometry, we deal with relations between the sides and the angles of a triangle.
2. Ratios of the sides of a right angled triangle with respect to its acute angles, are called trigonometric ratios of the angle.
3. For  $\angle A$ , AC is the base, BC the perpendicular and AB is the hypotenuse. For  $\angle B$ , BC is the base, AC the perpendicular and AB is the hypotenuse.



### 4. Six trigonometrical ratios



- (i)  $\text{sine } \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{y}{r}$ , sine  $\theta$  is written as  $\sin \theta$ .
- (ii)  $\text{cosine } \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{x}{r}$ , cosine  $\theta$  is written as  $\cos \theta$ .
- (iii)  $\text{tangent } \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$ , tangent  $\theta$  is written as  $\tan \theta$ .
- (iv)  $\text{cotangent } \theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$ , cotangent  $\theta$  is written as  $\cot \theta$ .
- (v)  $\text{secant } \theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{r}{x}$ , secant  $\theta$  is written as  $\sec \theta$ .
- (vi)  $\text{cosecant } \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{r}{y}$ , cosecant  $\theta$  is written as  $\text{cosec } \theta$ .

## 5. Relations between trigonometric ratios

### (a) Reciprocal relations

(i)  $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$  or  $\sin \theta = \frac{1}{\operatorname{cosec} \theta}$  or  $\sin \theta \operatorname{cosec} \theta = 1$

(ii)  $\sec \theta = \frac{1}{\cos \theta}$  or  $\cos \theta = \frac{1}{\sec \theta}$  or  $\cos \theta \sec \theta = 1$

(iii)  $\cot \theta = \frac{1}{\tan \theta}$  or  $\tan \theta = \frac{1}{\cot \theta}$  or  $\tan \theta \cot \theta = 1$

### (b) Quotient relations

(i)  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

(ii)  $\cot \theta = \frac{\cos \theta}{\sin \theta}$

6.  $\sin A$  is a symbol which denotes the ratio  $\frac{\text{perpendicular}}{\text{hypotenuse}}$ . It does not mean the product of  $\sin$  and  $A$  i.e.  $\sin A \neq \sin \times A$ . In fact  $\sin$  separated from  $A$  has no meaning. Similar interpretations follow for other trigonometric ratios.

7. Table of values of various trigonometric ratios  $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ .

T- ratios ↓ \ θ →	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Easier way to memorize the first row (values of sine ratio) as

sin	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
	$= 0$	$= \frac{1}{2}$	$= \frac{1}{\sqrt{2}}$	$= \frac{\sqrt{3}}{2}$	$= 1$

### 8. Trigonometric ratios of complementary angles

- (i)  $\sin(90^\circ - \theta) = \cos \theta$
- (ii)  $\cos(90^\circ - \theta) = \sin \theta$
- (iii)  $\tan(90^\circ - \theta) = \cot \theta$
- (iv)  $\cot(90^\circ - \theta) = \tan \theta$
- (v)  $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$
- (vi)  $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

### 9. Trigonometric Identities

(a) An equation involving trigonometric ratios of an angle  $\theta$  (say) is said to be a trigonometric identity, if it is satisfied for all values of  $\theta$  for which the given trigonometric ratios are defined.

(b) Some important trigonometric identities:

- (i)  $\sin^2 \theta + \cos^2 \theta = 1$   
 or  $\sin^2 \theta - 1 = -\cos^2 \theta$   
 or  $\cos^2 \theta - 1 = -\sin^2 \theta$
- (ii)  $\sec^2 \theta - \tan^2 \theta = 1$   
 or  $1 + \tan^2 \theta = \sec^2 \theta$   
 or  $\tan^2 \theta = \sec^2 \theta - 1$
- (iii)  $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$   
 or  $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$   
 or  $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$

(c) The following steps should be kept in mind while proving trigonometric identities:

- (i) Start with more complicated side of the identity and prove it equal to the other side.
- (ii) If the identity contains sine, cosine and other trigonometric ratios, then express all the ratios in terms of sine and cosine.
- (iii) If one side of an identity cannot be easily reduced to the other side value, then simplify both sides and prove them identically equal.
- (iv) While proving identities, never transfer terms from one side to another.