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QUADRATIC EQUATIONS
CLASS 10

Basic Concepts

1. An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation in x.
2. A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$, if $a\alpha^2 + b\alpha + c = 0$. Any quadratic equation can have at most two roots.

   Note: If α is a root of $ax^2 + bx + c = 0$, then we say that:

(i) $x = \alpha$ satisfies the equation $ax^2 + bx + c = 0$ or

(ii) $x = \alpha$ is a solution of the equation $ax^2 + bx + c = 0$

3. The roots of a quadratic equation $ax^2 + bx + c = 0$ are called the zeroes of the polynomial $ax^2 + bx + c = 0$
4. Solving a quadratic equation means finding its roots.
5. If $ax^2 + bx + c = 0$ can be factorised as $(x - \alpha)(x - \beta)$, then $ax^2 + bx + c = 0$ is equivalent to $(x - \alpha)(x - \beta) = 0 \Rightarrow x - \alpha = 0 \; or \; x - \beta = 0$ i.e., $x = \alpha$ or $x = \beta$.

   Here α and β are called the roots of the equation $ax^2 + bx + c = 0$.
6. To solve a quadratic equation by factorisation:

(a) Clear fractions and brackets, if necessary.

(b) Transfer all the terms to L.H.S. and combine like terms.

(c) Write the equation in the standard form, i.e., $ax^2 + bx + c = 0$.

(d) Factorise the L.H.S.

(e) Put each factor equal to zero and solve.

(f) Check each value by substituting it in the given equation.

7. The roots of a quadratic equation can also be found by using the method of completing the square.
8. The roots of the quadratic equation $ax^2 + bx + c = 0, a, b, c \in R$ and $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $(Sridharacharya's \; formula)$

The expression $b^2 - 4ac$ is called the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$.

9. The discriminant, usually denoted by D, decides the *nature of roots* of a quadratic equation.

(i) If D > 0, the equation the real roots and roots are unequal, i.e., unequal – real roots.
If D is a perfect square, the equation has unequal – rational roots.

(ii) If D = 0, the equation has real and equal roots and each root is $\frac{-b}{2a}$

(iii) If D < 0, the equation has no real roots.

10. (i) If $-p \geq 5$, then $p \leq -5$

(ii) If $-p \geq -5$, then $p \leq -5$

(iii) If $p^2 \geq 4$, then either $p \leq -2$ or $p \geq 2$

(iv) If $p^2 \leq 4$, then p lies between -2 and 2, i.e., $-2 \leq p \leq 2$

11. Quadratic equations can be applied to solve word problems involving various situations.
To solve problems leading to quadratic equations, following steps may be used:

(i) Represent the unknown quantity in the problem by a variable (letter).

(ii) Translate the problem into an equation involving this variable.

(iii) Solve the equation for the variable.

(iv) Check the result by satisfying the conditions of the original problem.

(v) A root of the quadratic equation, which does not satisfy the conditions of the problem, must be rejected.