Basic Concepts

1. An equation which can be put in the form $ax + by + c = 0$, where a, b and c are real numbers and a and b are not both zero, is called a linear equation in two variables x and y.
2. Every solution of the equation $ax + by + c = 0$ is a point on the line representing it. Or each solution (x, y) of a linear equation in two variables $ax + by + c = 0$, corresponds to a point on the line representing the equation and vice – versa.
3. A linear equation in two variables has an infinite number of solutions.
4. If we consider two equations of the form $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$, a pair of such equations is called a system of linear equations.
5. We have three types of systems of linear equations:

(i) **Independent System**, which has a unique solution. Such system is termed as a consistent system with unique solution.

(ii) **Inconsistent system**, which has no solution.

(iii) **Dependent system**, which represents a pair of equivalent equations and has an infinite number of solutions. Such system is also termed as a consistent system with infinite solutions.

6. A pair of linear equations in two variables which has a common point i.e., which has only one solution is called a consistent pair of linear equations.
7. A pair of linear equations in two variables which has no solution, i.e., the lines are parallel to each other is called an inconsistent pair of linear equations.
8. A pair of linear equations in two variables which are equivalent and has infinitely many solutions is called dependent pair of linear equations. Note that a dependent pair of linear equations is always consistent with infinite number of solutions.
9. If a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_{2y} + c_2 = 0$ represents

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(i) intersecting lines, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) parallel lines, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) coincident lines, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Converse of the above statement is also true.

10. **Graphical method of solving a pair of linear equations**

(a) To solve a system of linear equations graphically:

(i) Draw graph of the first equation.

(ii) On the same pair of axes, draw graph of the second equation.

(b) After representing a pair of linear equations graphically, only one of the following three possibilities can happen:

(i) The two lines will intersect at a point.

(ii) The two lines will be parallel.

(iii) The two lines will be coincident.

(c) (i) If the two lines intersect at a point, read the coordinates of the point of intersection to obtain the solution and verify your answer.

(ii) If the two lines are parallel, i.e., there is no point of intersection, write the system as inconsistent. Hence, no solution.

(iii) If the two lines have the same graph, then write the system as consistent with infinite number of solutions.

11. **Algebraic Methods of solving a pair of linear equations**

(a) Substitution Method:

(i) Suppose we are given two linear equations in x and y. For solving these equations by the substitution method, we proceed according to the following steps:

Step 1. Express y in terms of x in one of the given equations.

Step 2. Substitute this value of y in terms of x in the other equation. This gives a linear equation in x.

Step 3. Solve the linear equation in x obtained in step 2.

Step 4. Substitute this value of x in the relation taken in step 1 to obtain a linear equation in y.

Step 5. Solve the above linear equation in y to get the value of y.

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Note: We may interchange the role of x and y in the above method.

(ii) While solving a pair of linear equations, if we get the statements with no variables, we conclude as below.

(a) If the statement is true, we say that the equations have infinitely many solutions.

(b) If the statement is false, we say that the equations have no solution.

(iii) When the two given equations in x and y are such that the coefficients of x and y in one equation are interchanged in the other, then we add and subtract the two equations to get a pair of very simple equations.

(b) Elimination Method:

In this method, we eliminate one of the variables and proceed using the following steps.

Step 1. Multiply the given equations by suitable numbers to make the coefficients of one of the variables equal.

Step 2. If the equal coefficients are opposite in sign, then add the new equations, otherwise subtract them.

Step 3. The resulting equation is linear in one variable. Solve it to get the value of one of the unknown quantities.

Step 4. Substitute this value in any of the given equations.

Step 5. Solve it to get the value of the other variable.

(c) Cross Multiplication Method:

(i) The system of two linear equations $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$, where $\frac{a_1}{b_1} \neq \frac{b_1}{b_2}$ has a unique solution, given by

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

We generally write it as $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

The following diagram will help to apply the cross – multiplication method directly.

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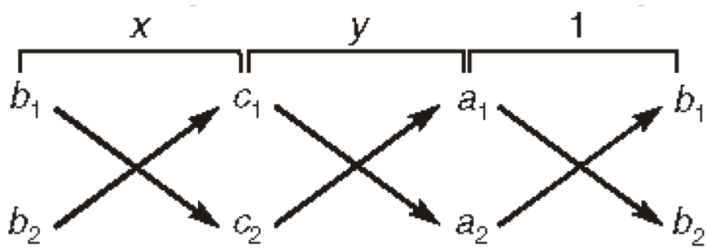

The arrows between the numbers indicate that they are to be multiplied. The products with upward arrows are to be subtracted from the products with downwards arrows.

(ii) The system of equations

$a_1x + b_1y + c_1 = 0$ ………(i)

$a_2x + b_2y + c_2 = 0$ ……...(ii)

(a) is consistent with unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., lines represented by equations (i) and (ii) intersect at a point.

(b) is inconsistent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, i.e., lines represented by equations (i) and (ii) are parallel.

(c) is consistent with infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, i.e., lines represented by equations (i) and (ii) are coincident.

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