

## Basic Concepts

1. Given positive integers  $a$  and  $b$ , there exist unique integers  $q$  and  $r$  satisfying  $a = bq + r$ ,  $0 \leq r < b$ . This result is known as **Euclid's division lemma**.
2. An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.
3. A lemma is a proven statement used for proving another statement.
4. HCF of two positive integers  $a$  and  $b$  is the largest positive integer  $d$  that divides  $a$  and  $b$ .
5. **Euclid's Division Algorithm:** To obtain the HCF of two positive integers, say  $c$  and  $d$  with  $c > d$ , we follow the steps below:  
**Step 1.** Apply Euclid's division lemma to find  $q$  and  $r$  where  $c = dq + r$ ,  $0 \leq r < d$ .  
**Step 2.** If  $r = 0$ , then,  $d$  is the HCF of  $c$  and  $d$ . If  $r \neq 0$ , then apply Euclid's division lemma to  $d$  and  $r$ .  
**Step 3.** Continue this process till the remainder is zero. The divisor at this stage will be the required HCF.
6. **The Fundamental Theorem of Arithmetic:** Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

**OR**

The prime factorisation of a natural number is unique, except for the order of its factors.

7. Any number which cannot be expressed in the form  $\frac{p}{q}$ , where  $p$  and  $q$  are integers and  $q \neq 0$  is called an irrational number.
8. Let  $p$  be a prime number. If  $p$  divides  $a^2$ , then  $p$  divides  $a$ , where  $a$  is a positive integer.
9. The sum or difference of a rational and an irrational number is irrational.

10. The product and quotient of a non – zero rational number and an irrational number is irrational.
11. Let  $x$  be a rational number whose decimal expansion terminates. Then  $x$  can be expressed in the form  $\frac{p}{q}$ , where  $p$  and  $q$  are coprime and the prime factorisation of  $q$  is of the form  $2^n 5^m$ , where  $n$  and  $m$  are non – negative integers.
12. Let  $x = \frac{p}{q}$  be a rational number, such that the prime factorisation of  $q$  is of the form  $2^n 5^m$ , where  $n$  and  $m$  are non negative integers. Then  $x$  has a decimal expansion which terminates.
13. If  $x = \frac{p}{q}$  is a rational number, such that the prime factorisation of  $q$  is of the form  $2^n 5^m$ , where  $m$  and  $n$  are whole numbers. If  $m = n$ , then the decimal expansion of  $x$  will terminate after  $m$  decimal places of decimal. If  $m > n$ , then the decimal expansion of  $x$  will terminate after  $m$  places of decimal. If  $n > m$ , then the decimal expansion of  $x$  will terminate after  $n$  places of decimal.
14. Let  $x = \frac{p}{q}$  be a rational number, such that the prime factorisation of  $q$  is not of the form  $2^n 5^m$ , where  $n$  and  $m$  are non – negative integers. Then  $x$  has a decimal expansion which is non – terminating repeating (recurring).
15. The decimal expansion of every rational number is either terminating or non – terminating repeating.