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ARITHMETIC PROGRESSION
ASSIGNMENT NO. 5 SOLUTIONS

1. $S_n = 4n^2 - 3n$
   $a_n = S_n - S_{n-1}$
   $a_n = 4n^2 - 3n - [4(n-1)^2 - 3(n-1)]$
   $a_n = 4n^2 - 3n - [4n^2 + 4 - 8n - 3n + 1]$
   $a_n = 4n^2 - 3n - 4n^2 + 11n - 5$
   $a_n = 8n - 5$

2. Let the numbers be $a - d, a, a + d$
   Sum = 27
   $a - d + a + a + d = 27$
   $3a = 27$
   $a = 9$
   Product = 405
   $(a - d)a(a + d) = 405$
   $(a^2 - d^2)a = 405$
   $(9^2 - d^2)9 = 405$
   $81 - d^2 = 45$
   $d^2 = 36$
   $d = \pm 6$
   When $a = 9, d = 6$
   $a - d = 9 - 6 = 3$
   $a = 9$
   $a + d = 9 + 6 = 15$
   When $a = 9, d = -6$
   $a - d = 9 - (-6) = 15$
   $a = 9$
   $a + d = 9 + (-6) = 3$
   Hence, the numbers are $3, 9$ and 15.

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3. Prove $S_{12} = 3(S_8 - S_4)$
   Let the first term be a and common difference be d.
   L.H.S. = $S_{12} = \frac{12}{2}(2a + 11d) = 6(2a + 11d) = 12a + 66d$
   R.H.S. = $3(S_8 - S_4)$
   $= 3\left[\frac{8}{2}(2a + 7d) - \frac{4}{2}(2a + 3d)\right]$
   $= 3[4(2a + 7d) - 2(2a + 3d)]$
   $= 3[8a + 28d - 4a - 6d]$
   $= 3(4a + 22d)$
   $= 12a + 66d$
   ∴ L.H.S. = R.H.S.
   Hence, $S_{12} = 3(S_8 - S_4)$

4. Let the first term be A and common difference be D.
   $a = A, b = A + D, c = A + 2d, d = A + 3D, e = A + 4d$
   $a - 4b + 6c - 4d + e$
   $= A - 4(A + D) + 6(A + 2D) - 4(A + 3D) + A + 4D$
   $= A - 4A - 4D + 6A + 12D - 4A - 12D + A + 4D$
   $= 8A - 8A + 16D - 16D$
   = 0

5. Prove $a_{3m+1} = 2a_{m+n+1}$
   Let a be the first term and d be the common difference.
   $a_{m+1} = 2a_{n+1}$
   $a + (m + 1 - 1)d = 2[a + (n + 1 - 1)d]$
   $a + md = 2(a + nd)$
   $a + md = 2a + 2nd$
   $md - 2nd = 2a - a$
   $d(m - 2n) = a \;\ldots\ldots(i)$
   L.H.S. = $a_{3m+1} = a + (3m + 1 - 1)d$
   $= a + 3md = d(m - 2n) + 3md$ {using equation (i)]
   $= md - 2nd + 3md = 4md - 2nd$
   R.H.S. = $2a_{m+n+1} = 2[a + (m + n + 1 - 1)d] = 2[a + (m + n)d]$
   = $2[d(m - 2n) + (m + n)d]$ [using equation (i)]
   = $2[md - 2nd + md + nd]$

$= 2[2md - nd]$
$= 4md - 2nd$
$\therefore$ L.H.S. = R.H.S.
$a_{3m+1} = 2a_{m+n+1}$

6. Let a be the first term and d be the common difference.
$a_p = q$
$a + (p-1)d = q \dots\dots (i)$
$a_q = p$
$a + (q-1)d = p \dots\dots (ii)$
Subtracting equation (ii) from (i)
$a + (p-1)d - [a + (q-1)d] = q - p$
$a + (p-1)d - a - (q-1)d = q - p$
$(p - 1 - q + 1)d = q - p$
$(p-q)d = -(p-q)$
$d = -1$
Put the value of d in equation (i)
$a + (p-1)(-1) = q$
$a - p + 1 = q$
$a = p + q - 1$
a_{p+q}
$= a + (p+q-1)d$
$= p + q - 1 + (p+q-1)(-1)$
$= p + q - 1 - p - q + 1$
$=0$
Hence, $(p+q)^{th}$ is 0.

7. Let a be the first term and d be the common difference
$S_m = S_n$
$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$
Multiplying both sides by 2
$2am + (m^2 - m)d = 2an + (n^2 - n)d$
$2am - 2an + (m^2 - m - n^2 + n)d = 0$
$2a(m-n) + [(m^2 - n^2) - (m-n)] = 0$
$2a(m-n) + [(m+n)(m-n) - (m-n)] = 0$

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Dividing both sides by $(m-n)$
$2a+(m-n-1)d=0 \ldots\ldots(i)$
S_{m+n}
$=\left(\frac{m+n}{2}\right)[2a+(m+n-1)d]$
$=\left(\frac{m+n}{2}\right)\times 0 \quad [using\ equation\ (i)]$
Hence, the sum of $(m+n)$ terms is 0.

8. Let a be the first term and d be the common difference.
$S_m = n$ (given)
$\frac{m}{2}[2a+(m-1)d]=n$
$2am+(m^2-m)d]=2n \ \ldots\ldots(i)$
$S_n = m$
$\frac{n}{2}[2a+(n-1)d]=m$
$2an+(n^2-n)d=2m \ \ \ldots\ldots(ii)$
Subtracting equation (ii) from (i)
$2am+(m^2-m)d-2an-(n^2-n)d=2n-2m$
$2a(m-n)+(m^2-m-n^2+n)d=2n-2m$
$2a(m-n)+[(m^2-n^2)-(m-n)]d=-2(m-n)$
$2a(m-n)+[(m+n)(m-n)-(m-n)d]=-2(m-n)$
Dividing both sides by $(m-n)$
$2a+(m+n-1)d=-2 \ \ldots\ldots(i)$

$S_{m+n}=\left(\frac{m+n}{2}\right)[2a+(m+n-1)d]$
$=\left(\frac{m+n}{2}\right)(-2)$ [using equation (i)]
$=-(m+n)$

9. Three digit numbers divisible by 5 leaving remainder 2 are 102, 107, 112, ……, 997
107 – 102 = 5
112 – 107 = 5
117 – 112 = 5

……………………
…………………….
Since $a_{k+1} - a_k$ is same for all the values of k
Hence, 102, 107, 112,…………, 997 form an A.P. with $a = 102$ and $d = 5$
Let number of terms be n.
$a_n = 997$
$a + (n-1)d = 997$
$102 + (n-1)5 = 997$
$(n-1)5 = 997 - 102$
$(n-1)5 = 895$
$n - 1 = 179$
$n = 180$
Hence, 180 three digit numbers are divisible by 5 leaving remainder 2.

10. Let a be the first term and d be the common difference.

$$a_m = \frac{1}{n}$$

$$a + (m-1)d = \frac{1}{n} \quad \ldots\ldots(i)$$

$$a_n = \frac{1}{m}$$

$$a + (n-1)d = \frac{1}{m} \quad \ldots\ldots(ii)$$

Subtracting equation (ii) from (i)

$$a + (m-1)d - a - (n-1)d = \frac{1}{n} - \frac{1}{m}$$

$$(m - 1 - n + 1)d = \frac{m-n}{mn}$$

$$(m-n)d = \frac{m-n}{mn}$$

$$d = \frac{1}{mn} \quad \ldots\ldots(iii)$$

Put the value of d in equation (i)

$$a + (m-1)\frac{1}{mn} = \frac{1}{n}$$

$$a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$$

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$a = \frac{1}{n} - \frac{1}{n} + \frac{1}{mn}$

$a = \frac{1}{mn}$ ……(iv)

$S_{mn} = \frac{mn}{2}[2a + (mn - 1)d]$

$= \frac{mn}{2}\left[2 \times \frac{1}{mn} + \frac{(mn-1)1}{mn}\right]$ $[using\ equation\ (iii)\ and\ (iv)]$

$= \frac{mn}{2} \times \frac{1}{mn}[2 + mn - 1]$

$= \frac{1}{2}(mn + 1)$

$Hence\ proved.$

11.Let a be the first term and d be the common term.

$S_1 = \frac{n}{2}[2a + (n - 1)d]$ ……(i)

$S_2 = \frac{2n}{2}[2a + (2n - 1)d]$ ……(ii)

$S_3 = \frac{3n}{2}[2a + (3n - 1)d]$ ……(iii)

R.H.S. = $3(S_2 - S_1)$

$= 3\left[\frac{2n}{2}\{2a + (2n - 1)d\} - \frac{n}{2}\{2a + (n - 1)d\}\right]$ $[using\ eq.(i)\ and\ (ii)]$

$= \frac{3n}{2}[2\{2a + (2n - 1)d - 2a - (n - 1)d]$

$= \frac{3n}{2}[4a + (4n - 2)d - 2a - (n - 1)d]$

$= \frac{3n}{2}[2a + (4n - 2 - n + 1)d]$

$= \frac{3n}{2}[2a + (3n - 1)d]$

$= S_3$ [using equation (iii)]

Hence, $S_3 = 3(S_2 - S_1)$

12.$a_n = a + nb$

$a_{n+1} = a + (n + 1)b$

$a_{n+1} - a_n$

$= a + (n + 1)b - (a + nb)$
$= a + (n + 1)b - a - nb$
$= nb + b - nb$
$= b$, which is independent of n
Hence, no matter what the real numbers a and b are, the sequence with n^{th} term $a + nb$ is always an A.P.
Common difference = b
$a_n = a + nb$
$a_1 = a + 1 \times b$
$a_1 = a + b$
$S_{20} = \frac{20}{2}[2a_1 + 19d]$
$S_{20} = 10[2(a + b) + 19b]$
$S_{20} = 10[2a + 2b + 19b]$
$S_{20} = 20a + 210b$

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