QUADRATIC EQUATIONS
ASSIGNMENT 16 SOLUTIONS

1. Let the original speed be x km/h
   Reduced speed = $(x - 100)$ km/h
   Distance = 3000 km
   Time taken at original speed = $\frac{distance}{speed} = \frac{3000}{x}$ hours
   Time taken at reduced speed = $\frac{3000}{x-100}$ hours
   According to the given condition
   $\frac{3000}{x-100} - \frac{3000}{x} = 1$
   $3000\left(\frac{1}{x-100} - \frac{1}{x}\right) = 1$
   $3000\left(\frac{x - x + 100}{(x-100)x}\right) = 1$
   $300000 = x^2 - 100x$
   $x^2 - 100x - 300000 = 0$
   $x^2 - 600x + 500x - 300000 = 0$
   $x(x - 600) + 500(x - 600) = 0$
   $(x - 600)(x + 500) = 0$
   Either $x - 600 = 0$ or $x + 500 = 0$
   $x = 600$ or $x = -500$
   Original speed = 600 km/h
   Original time = $\frac{3000}{600} = 5$ hours

2. Total amount = Rs. 800
   Let number of books = x
   New number of books = $x + 4$
   Cost of each book = Rs. $\frac{800}{x}$
   Cost of each book = Rs. $\frac{800}{x+4}$

$\frac{800}{x} - \frac{800}{x+4} = 10$

$800\left(\frac{1}{x} - \frac{1}{x+4}\right) = 10$

$80\left(\frac{x+4-x}{x(x+4)}\right) = 1$

$320 = x^2 + 4x$

$x^2 + 4x - 320 = 0$

$x^2 + 20x - 16x - 320 = 0$

$x(x+20) - 16(x+20) = 0$

$(x+20)(x-16) = 0$

Either $x + 20 = 0$ or $x - 16 = 0$

$x = -20$ or $x = 16$

We reject (-20) as number of books cannot be negative.

Hence, number of books = 16

3. Distance = 300 km

Let the usual speed = x km/h

Increased speed = $(x + 5)$ km/h

Time taken at usual speed = $\frac{distance}{speed} = \frac{300}{x}$ hours

Time taken at increased speed = $\frac{300}{x+5}$ hours

According to the given condition

$\frac{300}{x} - \frac{300}{x+5} = 2$

$300\left(\frac{1}{x} - \frac{1}{x+5}\right) = 2$

$150(x+5-x) = x(x+5)$

$750 = x^2 + 5x$

$x^2 + 5x - 750 = 0$

$x^2 + 30x - 25x - 750 = 0$

$(x+30)(x-25) = 0$

Either $x + 30 = 0$ or $x - 25 = 0$

$x = -30$ or $x = 25$

We reject (-30) as speed cannot be negative

$\therefore \quad x = 25$

Hence, usual speed of train = 25 km/h

4. Distance = 12 km
   Speed of boat in still water = 11 km/h
   Let the speed of stream = x km/h
   Speed while travelling downstream = $(11 + x)$ km/h
   Speed while travelling upstream = $(11 - x)$ km/h
   Time taken to travel 12 km downstream = $\frac{distance}{speed} = \frac{12}{11+x}$ hours
   Time taken to travel 12 km upstream = $\frac{12}{11-x}$ hours
   Total time = 2 hours 45 minutes = $2\ \frac{45}{60}$ hours = $2\frac{3}{4}$ hours
   According to the given condition
   $$\frac{12}{11+x} + \frac{12}{11-x} = 2\frac{3}{4}$$
   $$12\left(\frac{1}{11+x} + \frac{1}{11-x}\right) = \frac{11}{4}$$
   $$48(11 - x + 11 + x) = 11(11 + x)(11 - x)$$
   $$48 \times 22 = 11(121 - x^2)$$
   $$96 = 121 - x^2$$
   $$x^2 = 25$$
   $$x = \pm 5$$
   We reject (-5) as speed cannot be negative
   $\therefore\ x = 5$
   Hence, speed of stream = 5 km/h

5. Total amount = Rs. 1200
   Let cost of each book = Rs. x
   Increased price = Rs. $(x + 10)$
   Number of books bought at original price = $\frac{1200}{x}$
   Number of books bought at increased price = $\frac{1200}{x+10}$
   According to the given condition
   $$\frac{1200}{x} - \frac{1200}{x+10} = 10$$
   $$1200\left(\frac{1}{x} - \frac{1}{x+10}\right) = 10$$

$1200\left[\frac{x+10-x}{x(x+10)}\right]=10$

$1200 = x^2 + 10x$

$x^2 + 10x - 1200 = 0$

$x^2 + 40x - 30x - 1200 = 0$

$(x + 40)(x - 30) = 0$

$x = 30$ or (-40)

We reject (-40) as cost of book cannot be negative.

∴ Cost of each book = Rs. 30

6. Total number of apples = 300

Let number of students = x

Increased number of students = $x + 10$

Number of apples received by each student when number of students are $x = \frac{300}{x}$

Number of apples received by each student when number of students are $(x + 10) = \frac{300}{x+10}$

According to the given condition

$\frac{300}{x} - \frac{300}{x+10} = 1$

$300\left(\frac{1}{x} - \frac{1}{x+10}\right) = 1$

$300\left[\frac{x+10-x}{x(x+10)}\right] = 1$

$3000 = x^2 + 10x$

$x^2 + 10x - 3000 = 0$

$x^2 + 60x - 50x - 3000 = 0$

$x(x + 60) - 50(x + 60) = 0$

$(x + 60)(x - 50) = 0$

Either $x + 60 = 0$ or $x - 50 = 0$

$x = -60$ or $x = 50$

We reject (-60) as number of students cannot be negative.

$\therefore x = 50$

Hence the number of students = 50

7. Time = 30 minutes = $\frac{30}{60}$ hours = $\frac{1}{2}$ hours
   Let usual speed of aeroplane = x km/h
   Increased speed = $(x + 250)$ km/h
   Total distance = 1500 km
   Time taken at usual speed = $\frac{distance}{speed} = \frac{1500}{x}$ hours
   Time taken at increased speed = $\frac{1500}{x+250}$ hours
   According to the given condition
   $$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$
   $$1500\left(\frac{1}{x} - \frac{1}{x+250}\right) = \frac{1}{2}$$
   $$1500\left[\frac{x+250-x}{x(x+250)}\right] = \frac{1}{2}$$
   $$75000 = x^2 + 250x$$
   $$x^2 + 250x - 75000 = 0$$
   $$x^2 + 1000x - 750x - 75000 = 0$$
   $$(x + 1000)(x - 750) = 0$$
   Either $x + 1000 = 0$ or $x - 750 = 0$
   $x = -1000$ or 750
   We reject (-1000) as speed cannot be negative.
   $\therefore x = 750$
   Hence speed of aeroplane = 750 km/h

8. Total amount = Rs. 360
   Let number of days of tour = x
   Increased number of days = $(x + 4)$
   Daily expense for x days = Rs. $\frac{360}{x}$
   Daily expense for $(x + 4)$ days = Rs. $\frac{360}{x+4}$
   According to the given condition
   $$\frac{360}{x} - \frac{360}{x+4} = 3$$
   $$360\left(\frac{1}{x} - \frac{1}{x+4}\right) = 3$$

$120\left[\frac{x+4-x}{x(x+4)}\right]=1$

$480=x^2+4x$

$x^2+4x-480=0$

$x^2+24x-20x-480=0$

$x(x+24)-20(x+24)=0$

$(x+24)(x-20)=0$

Either $x+24=0$ or $x-20=0$

$x=20$ or -24

We reject (-24) as number of days cannot be negative.

$\therefore x=20$

Number of days = 20

9. Let average speed = x km/h

Time taken = $(x-1)$ hours

Distance = 30 km

Speed × time = 30

$x(x-1)=30$

$x^2-x-30=0$

$x^2-6x+5x-30=0$

$x(x-6)+5(x-6)=30$

$(x-6)(x+5)=0$

Either $x-6=0$ or $x+5-0$

$x=6$ or (-5)

Time taken = $6-1=5$ hours

10. Let age of mother = x years

Daughter's age = $(x-21)$ years

According to the given condition

$\frac{1}{12}[x(x-21)]=x-18$

$x^2-21x=12x-216$

$x^2-33x+216=0$

$x^2-24x-9x+216=0$

$(x-24)(x-9)=0$

Either $x-24=0$ or $x-9=0$

$x = 24$ or 9

We reject 9 as daughter's age for $x = 9$ will be $9 - 21 = -12$ which is not possible.

$\therefore x = 24$

Mother's age = 24 years

Daughter's age = 24 – 21 = 3 years