CBSEASSISTANCE.COM

POLYNOMIALS
ASSIGNMENT 39 SOLUTION

1. $(a+b+c)^2-(a-b+c)^2+(a+b-c)^2$
   $= a^2+b^2+c^2+2ab+2bc+2ca-a^2-b^2-c^2+2ab+2bc-2ca+a^2+b^2+c^2+2ab-2bc-2ca$
   $= a^2+b^2+c^2+6ab+2bc-2ca$

2. $x+\frac{1}{x}=9$ ……(i)
   Cubing both sides
   $x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=729$
   $x^3+\frac{1}{x^3}+3\times 9=729$ $[using\ equation\ (i)]$
   $x^3+\frac{1}{x^3}=729-27$
   $x^3+\frac{1}{x^3}=702$

3. $4(x^2+1)^2+13(x^2+1)-12$
   Put $x^2+1=y$
   $4y^2+13y-12$
   $= 4y^2+16y-3y-12$
   $= 4y(y+4)-3(y+4)$
   $= (y+4)(4y-3)$
   $= (x^2+1+4)[4(x^2+1)-3]$
   $= (x^2+5)(4x^2+1)$

4. $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

$= \left[(x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right)\right] - 2\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$

5. $\frac{6-2\sqrt{2}x-x^2}{2-x^2}$

$= \frac{6-3\sqrt{2}x+\sqrt{2}x-x^2}{\left(\sqrt{2}\right)^2-(x)^2}$

$= \frac{3\sqrt{2}\left(\sqrt{2}-x\right)+x\left(\sqrt{2}-x\right)}{\left(\sqrt{2}+x\right)\left(\sqrt{2}-x\right)}$

$= \frac{\left(\sqrt{2}-x\right)\left(3\sqrt{2}+x\right)}{\left(\sqrt{2}+x\right)\left(\sqrt{2}-x\right)}$

$= \frac{3\sqrt{2}+x}{\sqrt{2}+x}$

6. Put $x + y = a$ and $x - y = b$

$a - b = x + y - (x - y) = x + y - x + y = 2y$

$ab = (x + y)(x - y) = x^2 - y^2$

L.H.S.

$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

$= (x + y)^3 - (x - y)^3 - 3(x^2 - y^2)2y$

$= a^3 - b^3 - 3ab(a - b)$

$= (a - b)^3$

$= [x + y - (x - y)]^3$

$= (x + y - x + y)^3$

$= (2y)^3$

$= 8y^3$

= R.H.S.

Hence, $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = 8y^3$

7. $\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)$

$\left(x-\frac{1}{x}\right)^2 = 51-2 \quad [x^2+\frac{1}{x^2}=51\ given]$

$\left(x-\frac{1}{x}\right)^2 = 49$

Taking square root on both sides

$x-\frac{1}{x}=\pm 7$

When $x-\frac{1}{x}=7$

Cubing both sides

$x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=343$

$x^3-\frac{1}{x^3}-3\times 7=343$

$x^3-\frac{1}{x^3}=343+21$

$x^3-\frac{1}{x^3}=364$

When $x-\frac{1}{x}=-7$

Cubing both sides

$x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=-343$

$x^3-\frac{1}{x^3}-3(-7)=-343$

$x^3-\frac{1}{x^3}=-343-21$

$x^3-\frac{1}{x^3}=-364$

8. $x+\frac{1}{x}=5$

Squaring both sides

$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=25$

$x^2+\frac{1}{x^2}=25-2$

$x^2+\frac{1}{x^2}=23$

Cubing both sides

$(x^2)^3+\left(\frac{1}{x^2}\right)^3+3(x^2)\left(\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)=12167$

$x^6+\frac{1}{x^6}+3\times 23=12167$

$x^6+\frac{1}{x^6}=12167-69$

$x^6+\frac{1}{x^6}=12098$

9. Let $p(x)=2x^4-8x^3+3x^2+12x-9$

x^2-4x+3

$=x^2-3x-x+3$

$=x(x-3)-1(x-3)$

$=(x-3)(x-1)$

Put $x=3$

$p(3)=2(3)^4-8(3)^3+3(3)^2+12(3)-9$

$=162-216+27+36-9$

$=225-225$

$=0$

By factor theorem, $(x-3)$ is a factor of $p(x)$

Put $x=1$

$p(1)=2(1)^4-8(1)^3+3(1)^2+12(1)-9$

$=2-8+3+12-9$

$=17-17$

$=0$

By factor theorem, $(x-1)$ is a factor of $p(x)$

Since $(x-3)$ and $(x-1)$ are factors of $p(x)$

$\therefore (x-3)(x-1)$ is a factor of $p(x)$
$= x^2 - 4x + 3$ is a factor of $p(x)$

10. $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$
By remainder theorem
$f(1) = 5$
$(1)^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$
$1 - 2 + 3 - a + b = 5$
$2 - a + b = 5$
$-a + b = 3 \dots\dots\dots (i)$
By remainder theorem
$f(-1) = 19$
$(-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$
$1 + 2 + 3 + a + b = 19$
$a + b = 13 \dots\dots\dots (ii)$
Adding equation (i) and (ii)
$2b = 16$
$b = 8$
From equation (ii)
$a + 8 = 13$
$a = 5$