REAL NUMBERS

SOLUTION 2

If a and b are two odd positive integers such that $a > b$, then prove that one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

Solution:

Let $a = 4m + 1$ and $b = 4n + 1$

$$\frac{a+b}{2} = \frac{(4m+1)+(4n+1)}{2} = \frac{4m+4n+2}{2} = \frac{2(2m+2n+1)}{2}$$

$= 2m + 2n + 1 = 2(m + n) + 1 = 2q + 1, where\ q = m + n$, which is odd

$$\frac{a-b}{2} = \frac{(4m+1)-(4n+1)}{2} = \frac{4m+1-4n-1}{2} = \frac{4m-4n}{2} = \frac{4(m-n)}{2}$$

$= 2(m - n) = 2q, where\ q = m - n$, which is even.

∴ If a and b are two odd positive integers such that $a > b$, then one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

(Note: This question can also be solved by taking $a = 2m + 1$ and $b = 2n + 1$)