

Prove that the product of two consecutive positive integers is divisible by 2.

Solution:

Let $(n - 1)$ and n be two positive integers.

Their product $n(n - 1) = n^2 - n$

Any positive integer is of the form $2q$ or $2q + 1$, for some integer q .

The following cases arise:

Case 1

When $n = 2q$

$$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q - 1)$$

$$n^2 - n = 2m, \text{ where } m = q(2q - 1)$$

$\therefore n^2 - n$ is divisible by 2.

Case 2

When $n = 2q + 1$

$$n^2 - n = (2q + 1)^2 - (2q + 1) = (2q + 1)(2q + 1 - 1) = 2q(2q + 1)$$

$$n^2 - n = 2m, \text{ where } m = q(2q + 1)$$

$\therefore n^2 - n$ is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n .

$(n - 1)n$ is divisible by 2.

\therefore The product of two consecutive positive integers is divisible by 2.