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REAL NUMBERS

SOLUTION 8

Prove that if a positive integer is of the form $6q + 5$, then it is of the form $3q + 2$ for some integer q, but not conversely.

Solution:

Let $n = 6q + 5$, where q is a positive integer.

We know that any positive integer is of the form $3k$ or, $3k + 1$ or, $3k + 2$

$\therefore q = 3k$ or, $3k + 1$ or, $3k + 2$

If $q = 3k$, then

$n = 6(3k) + 5$

$= 18k + 5$

$= 18k + 3 + 2$

$= 3(6k + 1) + 2 = 3m + 2$, where $m = 6k + 1$

If $q = 3k + 1$, then

$n = 6(3k + 1) + 5$

$= 18k + 6 + 5$

$= 18k + 11$

$= 18k + 9 + 2$

$= 3(6k + 3) + 2$

$= 3m + 2$, where $m = 6k + 3$

If $q = 3k + 2$, then

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$n = 6(3k + 2) + 5$

$= 18k + 12 + 5$

$= 18k + 17$

$= 18k + 15 + 2$

$= 3(6k + 5) + 2$

$= 3m + 2$, where $m = 6k + 5$

$\therefore$ If a positive integer is of the form $6q + 5$, then it is of the form $3q + 2$.

Now let $n = 3q + 2$, where q is a positive integer.

We know that any positive integer is of the form $6q, 6q + 1, 6q + 2, 6q + 3, 6q + 4, 6q + 5$

Now, if $q = 6q$

$n = 3q + 2$

$n = 3(6q) + 2$

$n = 18q + 2$

$n = 2(9q + 1)$

$n = 2m$, where $m = 9q + 1$

Hence, $3q + 2$ is not of the form $6q + 5$.

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