REAL NUMBERS

SOLUTION 6

Prove that the square of any positive integer is of the form $4q$ or $4q + 1$, for some integer q.

Solution:

Let a be any positive integer and $b = 2$

By Euclid's division algorithm

$a = 2m + r, 0 \leq r < 2$

When $r = 0, a = 2m$

Squaring both sides

$a^2 = (2m)^2 = 4m^2 = 4q$, where $q = m^2$

When $r = 1, a = 2m + 1$

Squaring both sides

$a^2 = (2m + 1)^2$

$= (2m)^2 + 2(2m)(1) + (1)^2$

$= 4m^2 + 4m + 1$

$= 4(m^2 + m) + 1$

$= 4q + 1$, where $q = m^2 + m$

Hence, the square of any positive integer is of the form $4q$ or $4q + 1$ for some integer q.