# REAL NUMBERS

## SOLUTION 5

**Prove that the square of any positive integer is of the form $3m$ or, $3m + 1$ but not of the form $3m + 2$.**

**Solution:**

Let $a$ be any positive integer and $b = 3$.

By Euclid's division algorithm, $a = 3q + r, 0 \leq r < 3$

Possible values of $r = 0, 1, 2$

When $r = 0, a = 3q$

Squaring both sides

$a^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$, where $m = 3q^2$

When $r = 1, a = 3q + 1$

Squaring both sides

$a^2 = (3q + 1)^2$

$= (3q)^2 + 2(3q)(1) + (1)^2$

$= 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1, m = 3q^2 + 2q$

When $r = 2, a = 3q + 2$

Squaring both sides

$a^2 = (3q + 2)^2$

$= (3q)^2 + 2(3q)(2) + (2)^2$

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$, where $m = 3q^2 + 4q + 1$

Hence, the square of any positive integer is of the form $3m$ or, $3m + 1$ but not of the form $3m + 2$.