REAL NUMBERS

SOLUTION 4

Show that any positive odd integer is of the form $6q + 1$ or $6q + 3$ or, $6q + 5$, where q is some integer.

Solution:

Let a be any positive integer and $b = 6$.

By Euclid's division algorithm, $a = 6q + r, 0 \leq r < 6$

Possible values of $r = 0, 1, 2, 3, 4, 5$

When $r = 0,\ a = 6q = 2(3q) = 2m$, where $m = 3q$, which is even.

When $r = 1, a = 6q + 1 = 2(3q) + 1 = 2m + 1$, where $m = 3q$, which is odd.

When $r = 2, a = 6q + 2 = 2(3q + 1) = 2m$, where $m = 3q + 1$, which is even.

When $r = 3, a = 6q + 3 = 6q + 2 + 1 = 2(3q + 1) + 1 = 2m + 1$, where $m = 3q + 1$, which is odd.

When $r = 4, a = 6q + 4 = 2(3q + 2) = 2m$, where $m = 3q + 2$, which is even.

When $r = 5, a = 6q + 5 = 6q + 4 + 1 = 2(3q + 2) + 1 = 2m + 1$, where $m = 3q + 2$, which is odd

Hence, any positive odd integer is of the form $6q + 1$ or, $6q + 3$ or, $6q + 5$, where q is some integer.