REAL NUMBERS

SOLUTION 3

Show that the square of an odd positive integer is of the form $8q + 1$, for some integer q.

Solution:

Any odd positive integer is of the form $4m + 1$ or $4m + 3$

$(4m + 1)^2$

$= (4m)^2 + 2(4m)(1) + (1)^2$

$= 16m^2 + 8m + 1$

$= 8(2m^2 + m) + 1$

$= 8q + 1$, where $q = 2m^2 + m$

$(4m + 3)^2$

$= (4m)^2 + 2(4m)(2) + (3)^2$

$= 16m^2 + 16m + 9$

$= 16m^2 + 16m + 8 + 1$

$= 8(2m^2 + 2m + 1) + 1$

$= 8q + 1$, where $q = 2m^2 + 2m + 1$

Hence, the square of an odd positive integer is of the form $8q + 1$, for some integer q.