# REAL NUMBERS

## SOLUTION 11

**For any positive integer $n$, prove that $n^3 - n$ is divisible by 6.**

**Solution:**

$n^3 - n = n(n^2 - 1) = n(n+1)(n-1)$

Any positive integer is of the form $6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

When $n = 6q$

$(n-1)n(n+1)$

$= (6q-1)6q(6q+1)$

$= 6q(6q-1)(6q+1)$

$= 6m$, where $m = q(6q-1)(6q+1)$

When $n = 6q+1$

$(n-1)n(n+1)$

$= (6q+1-1)(6q+1)(6q+1+1)$

$= 6q(6q+1)(6q+2) = 6m$, where $m = q(6q+1)(6q+2)$

When $n = 6q+2$

$(n-1)n(n+1)$

$= (6q+2-1)(6q+2)(6q+2+1)$

$= (6q+1)(6q+2)(6q+3)$

$= 6(6q+1)(3q+1)(2q+1) = 6m$, where $m = (6q+1)(3q+1)(2q+1)$

When $n = 6q + 3$

$(n - 1)n(n + 1)$

$= (6q + 3 - 1)(6q + 3)(6q + 3 + 1)$

$= (6q + 2)(6q + 3)(6q + 4)$

$= 6(3q + 1)(2q + 1)(6q + 4) = 6m$, where $m = (3q + 1)(2q + 1)(6q + 4)$

When $n = 6q + 4$

$(n - 1)n(n + 1)$

$= (6q + 4 - 1)(6q + 4)(6q + 4 + 1)$

$= (6q + 3)(6q + 4)(6q + 5)$

$= 6(2q + 1)(3q + 2)(6q + 5) = 6m$, where $m = (2q + 1)(3q + 2)(6q + 5)$

When $n = 6q + 5$

$(n - 1)n(n + 1)$

$= (6q + 5 - 1)(6q + 5)(6q + 5 + 1)$

$= (6q + 4)(6q + 5)(6q + 6)$

$= 6(6q + 4)(6q + 5)(q + 1) = 6m$, where $m = (6q + 4)(6q + 5)(q + 1)$

Hence, for any positive integer $n$, prove that $n^3 - n$ divisible by 6.