REAL NUMBERS

SOLUTION 10

Prove that the product of three consecutive positive integers is divisible by 6.

Solution:

Let three consecutive integers be $n-1, n, n+1$,

Any positive integer is of the form $6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

When $n = 6q$

$(n-1)n(n+1)$

$= (6q-1)6q(6q+1)$

$= 6q(6q-1)(6q+1)$

$= 6m$, where $m = q(6q-1)(6q+1)$

When $n = 6q+1$

$(n-1)n(n+1)$

$= (6q+1-1)(6q+1)(6q+1+1)$

$= 6q(6q+1)(6q+2) = 6m$, where $m = q(6q+1)(6q+2)$

When $n = 6q+2$

$(n-1)n(n+1)$

$= (6q+2-1)(6q+2)(6q+2+1)$

$= (6q+1)(6q+2)(6q+3)$

$= 6(6q+1)(3q+1)(2q+1) = 6m$, where $m = (6q+1)(3q+1)(2q+1)$

When $n = 6q+3$

$(n-1)n(n+1)$

$= (6q+3-1)(6q+3)(6q+3+1)$

$= (6q+2)(6q+3)(6q+4)$

$= 6(3q+1)(2q+1)(6q+4) = 6m$, where $m = (3q+1)(2q+1)(6q+4)$

When $n = 6q+4$

$(n-1)n(n+1)$

$= (6q+4-1)(6q+4)(6q+4+1)$

$= (6q+3)(6q+4)(6q+5)$

$= 6(2q+1)(3q+2)(6q+5) = 6m$, where $m = (2q+1)(3q+2)(6q+5)$

When $n = 6q+5$

$(n-1)n(n+1)$

$= (6q+5-1)(6q+5)(6q+5+1)$

$= (6q+4)(6q+5)(6q+6)$

$= 6(6q+4)(6q+5)(q+1) = 6m$, where $m = (6q+4)(6q+5)(q+1)$

Therefore, the product of three consecutive positive integer is divisible by 6.