Ex. 6 D

1. Find each of the following products:

(i) $(x+6)(x+6)$

Solution:

$(x+6)(x+6)$

$= (x+6)^2$

$= x^2 + 2 \times x \times 6 + 6^2 \qquad [(a+b)^2 = a^2 + 2ab + b^2]$

$= x^2 + 12x + 36$

(ii) $(4x+5y)(4x+5y)$

Solution:

$(4x+5y)(4x+5y)$

$= (4x+5y)^2$

$= (4x)^2 + 2 \times 4x \times 5y + (5y)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 16x^2 + 40xy + 25y^2$

(iii) $(7a+9b)(7a+9b)$

Solution:

$(7a+9b)(7a+9b)$

$= (7a+9b)^2$

$= (7a)^2 + 2 \times 7a \times 9b + (9b)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 49a^2 + 126ab + 81b^2$

(iv) $\left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x + \frac{4}{5}y\right)$

Solution:

$\left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x + \frac{4}{5}y\right)$

$= \left(\frac{2}{3}x + \frac{4}{5}y\right)^2$

$= \left(\frac{2}{3}x\right)^2 + 2\left(\frac{2}{3}x\right)\left(\frac{4}{5}y\right) + \left(\frac{4}{5}y\right)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$

$= \frac{4}{9}x^2 + \frac{16}{15}xy + \frac{16}{25}y^2$

(v) $(x^2 + 7\,)(x^2 + 7)$

Solution:

$(x^2 + 7)(x^2 + 7)$

$= (x^2 + 7)^2$

$= (x^2)^2 + 2(x^2)(7) + (7)^2 \qquad [(a+b)^2 = a^2 + 2ab + b^2]$

$= x^4 + 14x^2 + 49$

(vi) $\left(\frac{5}{6}a^2 + 2\right)\left(\frac{5}{6}a^2 + 2\right)$

Solution:

$\left(\frac{5}{6}a^2 + 2\right)\left(\frac{5}{6}a^2 + 2\right)$

$= \left(\frac{5}{6}a^2 + 2\right)^2$

$= \left(\frac{5}{6}a^2\right)^2 + 2\left(\frac{5}{6}a^2\right)(2) + (2)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$

$= \frac{25}{36}a^4 + \frac{10}{3}a^2 + 4$

2. Find each of the following products:

(i) $(x - 4)(x - 4)$

Solution:

$(x - 4)(x - 4)$

$= (x - 4)^2$

$= x^2 - 2(x)(4) + 4^2 \qquad [(a-b)^2 = a^2 - 2ab + b^2]$

$= x^2 - 8x + 16$

(ii) $(2x - 3y)(2x - 3y)$

Solution:

$(2x - 3y)(2x - 3y)$

$= (2x - 3y)^2$

$= (2x)^2 - 2(2x)(3y) + (3y)^2 \qquad [(a-b)^2 = a^2 - 2ab + b^2]$

$= 4x^2 - 12xy + 9y^2$

(iii) $\left(\frac{3}{4}x - \frac{5}{6}y\right)\left(\frac{3}{4}x - \frac{5}{6}y\right)$

Solution:

$$\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)$$
$$=\left(\frac{3}{4}x-\frac{5}{6}y\right)^2$$
$$=\left(\tfrac{3}{4}x\right)^2-2\left(\tfrac{3}{4}x\right)\left(\tfrac{5}{6}y\right)+\left(\tfrac{5}{6}y\right)^2\ [(a-b)^2=a^2-2ab+b^2]$$
$$=\frac{9}{16}x^2-\frac{5}{4}xy+\frac{25}{36}y^2$$

(iv) $\left(x-\frac{3}{x}\right)\left(x-\frac{3}{x}\right)$

Solution:

$$\left(x-\frac{3}{x}\right)\left(x-\frac{3}{x}\right)$$
$$=\left(x-\frac{3}{x}\right)^2$$
$$=(x)^2-2(x)\left(\tfrac{3}{x}\right)+\left(\tfrac{3}{x}\right)^2 \qquad [(a-b)^2=a^2-2ab+b^2]$$
$$=x^2-6+\frac{9}{x^2}$$

(v) $\left(\frac{1}{3}x^2-9\right)\left(\frac{1}{3}x^2-9\right)$

Solution:

$$\left(\frac{1}{3}x^2-9\right)\left(\frac{1}{3}x^2-9\right)$$
$$=\left(\frac{1}{3}x^2-9\right)^2$$
$$=\left(\tfrac{1}{3}x^2\right)^2-2\left(\tfrac{1}{3}x^2\right)(9)+(9)^2 \quad [(a-b)^2=a^2-2ab+b^2]$$
$$=\frac{1}{9}x^4-6x^2+81$$

(vi) $\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(\frac{1}{2}y^2-\frac{1}{3}y\right)$

Solution:

$$\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(\frac{1}{2}y^2-\frac{1}{3}y\right)$$
$$=\left(\frac{1}{2}y^2-\frac{1}{3}y\right)^2$$
$$=\left(\frac{1}{2}y^2\right)^2-2\left(\frac{1}{2}y^2\right)\left(\frac{1}{3}y\right)+\left(\frac{1}{3}y\right)^2\ [(a-b)^2=a^2-2ab+b^2]$$

$$= \frac{1}{4}y^4 - \frac{1}{3}y^3 + \frac{1}{9}y^2$$

3. Expand:

(i) $(8a + 3b)^2$

Solution:

$(8a + 3b)^2$

$= (8a)^2 + 2(8a)(3b) + (3b)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2]$

$= 64a^2 + 48ab + 9b^2$

(ii) $(7x + 2y)^2$

Solution:

$(7x + 2y)^2$

$= (7x)^2 + 2(7x)(2y) + (2y)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2]$

$= 49x^2 + 28xy + 4y^2$

(iii) $(5x + 11)^2$

Solution:

$(5x + 11)^2$

$(5x)^2 + 2(5x)(11) + (11)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2]$

$= 25x^2 + 110x + 121$

(iv) $\left(\frac{a}{2} + \frac{2}{a}\right)^2$

Solution:

$\left(\frac{a}{2} + \frac{2}{a}\right)^2$

$= \left(\frac{a}{2}\right)^2 + 2\left(\frac{a}{2}\right)\left(\frac{2}{a}\right) + \left(\frac{2}{a}\right)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2]$

$= \frac{a^2}{4} + 2 + \frac{4}{a^2}$

(v) $\left(\frac{3x}{4} + \frac{2y}{9}\right)^2$

Solution:

$\left(\frac{3x}{4} + \frac{2y}{9}\right)^2$

$= \left(\frac{3x}{4}\right)^2 + 2\left(\frac{3x}{4}\right)\left(\frac{2y}{9}\right) + \left(\frac{2y}{9}\right)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2]$

$$= \frac{9x^2}{16} + \frac{xy}{3} + \frac{4y^2}{81}$$

(vi) $(9x - 10)^2$

Solution:

$(9x - 10)^2$

$= (9x)^2 - 2(9x)(10) + (10)^2 \quad [(a-b)^2 = a^2 - 2ab + b^2]$

$= 81x^2 - 180x + 100$

(vii) $(x^2y - yz^2)^2$

Solution:

$(x^2y - yz^2)^2$

$= (x^2y)^2 - 2(x^2y)(yz^2) + (yz^2)^2 \quad [(a-b)^2 = a^2 - 2ab + b^2]$

$= x^4y^2 - 2x^2y^2z^2 + y^2z^4$

(viii) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$

Solution:

$\left(\frac{x}{y} - \frac{y}{x}\right)^2$

$= \left(\frac{x}{y}\right)^2 - 2\left(\frac{x}{y}\right)\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 \quad [(a-b)^2 = a^2 - 2ab + b^2]$

$= \frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$

$= \frac{x^2}{y^2} + \frac{y^2}{x^2} - 2$

(ix) $\left(3m - \frac{4}{5}n\right)^2$

Solution:

$\left(3m - \frac{4}{5}n\right)^2$

$= (3m)^2 - 2(3m)\left(\frac{4}{5}n\right) + \left(\frac{4}{5}n\right)^2 \quad [(a-b)^2 = a^2 - 2ab + b^2]$

$= 9m^2 - \frac{24}{5}mn + \frac{16}{25}n^2$

4. Find each of the following products:

(i) $(x + 3)(x - 3)$

Solution:

$(x+3)(x-3)$

$= x^2 - 3^2$ $\qquad [(a+b)(a-b) = a^2 - b^2]$

$= x^2 - 9$

(ii) $(2x+5)(2x-5)$

Solution:

$(2x+5)(2x-5)$

$= (2x)^2 - (5)^2$ $\qquad [(a+b)(a-b) = a^2 - b^2]$

$= 4x^2 - 25$

(iii) $(8+x)(8-x)$

Solution:

$(8+x)(8-x)$

$= 8^2 - x^2$ $\qquad [(a+b)(a-b) = a^2 - b^2]$

$= 64 - x^2$

(iv) $(7x+11y)(7x-11y)$

Solution:

$(7x+11y)(7x-11y)$

$= (7x)^2 - (11y)^2$ $\qquad [(a+b)(a-b) = a^2 - b^2]$

$= 49x^2 - 121y^2$

(v) $\left(5x^2 + \frac{3}{4}y^2\right)\left(5x^2 - \frac{3}{4}y^2\right)$

Solution:

$\left(5x^2 + \frac{3}{4}y^2\right)\left(5x^2 - \frac{3}{4}y^2\right)$

$= (5x^2)^2 - \left(\frac{3}{4}y^2\right)^2$ $\qquad [(a+b)(a-b) = a^2 - b^2]$

$= 25x^4 - \frac{9}{16}y^4$

(vi) $\left(\frac{4x}{5} - \frac{5y}{3}\right)\left(\frac{4x}{5} + \frac{5y}{3}\right)$

Solution:

$\left(\frac{4x}{5} - \frac{5y}{3}\right)\left(\frac{4x}{5} + \frac{5y}{3}\right)$

$= \left(\frac{4x}{5}\right)^2 - \left(\frac{5y}{3}\right)^2$ $\qquad [(a+b)(a-b) = a^2 - b^2]$

$= \frac{16}{25}x^2 - \frac{25}{9}y^2$

(vii) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

Solution:

$\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= (x)^2 - \left(\frac{1}{x}\right)^2$ $\quad [(a+b)(a-b) = a^2 - b^2]$

$= x^2 - \frac{1}{x^2}$

(viii) $\left(\frac{1}{x} + \frac{1}{y}\right)\left(\frac{1}{x} - \frac{1}{y}\right)$

Solution:

$\left(\frac{1}{x} + \frac{1}{y}\right)\left(\frac{1}{x} - \frac{1}{y}\right)$

$= \left(\frac{1}{x}\right)^2 - \left(\frac{1}{y}\right)^2$ $\quad [(a+b)(a-b) = a^2 - b^2]$

$= \frac{1}{x^2} - \frac{1}{y^2}$

(ix) $\left(2a + \frac{3}{b}\right)\left(2a - \frac{3}{b}\right)$

Solution:

$\left(2a + \frac{3}{b}\right)\left(2a - \frac{3}{b}\right)$

$= (2a)^2 - \left(\frac{3}{b}\right)^2$ $\quad [(a+b)(a-b) = a^2 - b^2]$

$= 4a^2 - \frac{9}{b^2}$

5. Using the formula for squaring a binomial, evaluate the following:

(i) $(54)^2$

Solution:

$(54)^2$

$= (50 + 4)^2$

$= (50)^2 + 2(50)(4) + (4)^2$ $\quad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 2500 + 400 + 16$

$= 2916$

(ii) $(82)^2$

Solution:

$(82)^2$

$= (80 + 2)^2$

$= (80)^2 + 2(80)(2) + (2)^2 \qquad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 6400 + 320 + 4$

$= 6724$

(iii) $(103)^2$

Solution:

$(103)^2$

$= (100 + 3)^2$

$= (100)^2 + 2(100)(3) + (3)^2 \qquad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 10000 + 600 + 9$

$= 10609$

(iv) $(704)^2$

Solution:

$(704)^2$

$= (700 + 4)^2$

$= (700)^2 + 2(700)(4) + (4)^2 \qquad [(a+b)^2 = a^2 + 2ab + b^2]$

$= 490000 + 5600 + 16$

$= 495616$

6. Using the formula for squaring a binomial, evaluate the following:

(i) $(69)^2$

Solution:

$(69)^2$

$= (70 - 1)^2$

$= (70)^2 - 2(70)(1) + (1)^2 \qquad [(a-b)^2 = a^2 - 2ab + b^2]$

$= 4900 - 140 + 1$

$= 4761$

(ii) $(78)^2$

Solution:

$(78)^2$

$= (80 - 2)^2$

$= (80)^2 - 2(80)(2) + (2)^2 \qquad [(a-b)^2 = a^2 - 2ab + b^2]$

$= 6400 - 320 + 4$
$= 6084$

(iii) $(197)^2$
Solution:
$(197)^2$
$= (200 - 3)^2$
$= (200)^2 - 2(200)(3) + (3)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$
$= 40000 - 1200 + 9$
$= 38809$

(iv) $(999)^2$
Solution:
$(999)^2$
$= (1000 - 1)^2$
$= (1000)^2 - 2(1000)(1) + (1)^2 \quad [(a+b)^2 = a^2 + 2ab + b^2]$
$= 1000000 - 2000 + 1$
$= 998001$

7. Find the value of:
(i) $(82)^2 - (18)^2$
Solution:
$(82)^2 - (18)^2$
$= (82 + 18)(82 - 18) \quad [(a+b)(a-b) = a^2 - b^2]$
$= 100 \times 64$
$= 6400$

(ii) $(128)^2 - (72)^2$
Solution:
$(128)^2 - (72)^2$
$= (128 + 72)(128 - 72) \quad [(a+b)(a-b) = a^2 - b^2]$
$= 200 \times 56$
$= 11200$

(iii) 197×203
Solution:
197×203
$= (200 - 3)(200 + 3)$

$= (200)^2 - (3)^2$ $[(a+b)(a-b) = a^2 - b^2]$
$= 40000 - 9$
$= 39991$

(iv) $\frac{198\times198-102\times102}{96}$

Solution:

$$\frac{198 \times 198 - 102 \times 102}{96}$$
$$= \frac{(198)^2 - (102)^2}{96}$$
$= \frac{(198+102)(198-102)}{96}$ $[(a+b)(a-b) = a^2 - b^2]$
$$= \frac{300 \times 96}{96}$$
$= 300$

(v) 14.7×15.3

Solution:

14.7×15.3
$= (15 - 0.3)(15 + 0.3)$
$= (15)^2 - (0.3)^2$ $[(a+b)(a-b) = a^2 - b^2]$
$= 225 - 0.09$
$= 224.91$

(vi) $(8.63)^2 - (1.37)^2$

Solution:

$(8.63)^2 - (1.37)^2$
$= (8.63 + 1.37)(8.63 - 1.37)$ $[(a+b)(a-b) = a^2 - b^2]$
$= 10 \times 7.26$
$= 72.6$

8. Find the value of the expression $(9x^2 + 24x + 16)$, when $x = 12$

Solution:

$9x^2 + 24x + 16$
$= (3x)^2 + 2 \times 3x \times 4 + (4)^2$
$= (3x + 4)^2$ $[a^2 + 2ab + b^2 = (a+b)^2]$
Put $x = 12$
$(3 \times 12 + 4)^2$
$= (36 + 4)^2$

$= 40^2$
$= 1600$

9. Find the value of the expression $(64x^2 + 81y^2 + 144xy)$, when $x = 11$ and $y = \frac{4}{3}$

Solution:

$64x^2 + 81y^2 + 144xy$
$= (8x)^2 + (9y)^2 + 2 \times 8x \times 9y$
$= (8x + 9y)^2$ $\quad [a^2 + 2ab + b^2 = (a + b)^2]$

Put $x = 11$ and $y = \frac{4}{3}$

$\left(8 \times 11 + 9 \times \frac{4}{3}\right)^2$
$= (88 + 12)^2$
$= (100)^2$
$= 10000$

10. Find the value of the expression $(36x^2 + 25y^2 - 60xy)$, when $x = \frac{2}{3}$ and $y = \frac{1}{5}$

Solution:

$36x^2 + 25y^2 - 60xy$
$= (6x)^2 + (5y)^2 - 2(6x)(5y)$
$= (6x - 5y)^2$ $\quad [a^2 - 2ab + b^2 = (a - b)^2]$

Put $x = \frac{2}{3}$ and $y = \frac{1}{5}$

$\left(6 \times \frac{2}{3} - 5 \times \frac{1}{5}\right)^2$
$= (4 - 1)^2$
$= 3^2$
$= 9$

11. If $\left(x + \frac{1}{x}\right) = 4$, find the values of

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

Solution:

(i) $x + \frac{1}{x} = 4$

Squaring both sides

$$\left(x + \frac{1}{x}\right)^2 = 4^2$$

$$x^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) = 16 \qquad [a^2 + 2ab + b^2 = (a+b)^2]$$

$$x^2 + \frac{1}{x^2} + 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 - 2$$

$$x^2 + \frac{1}{x^2} = 14$$

(ii) Squaring both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (14)^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) = 196 \qquad [a^2 + 2ab + b^2 = (a+b)^2]$$

$$x^4 + \frac{1}{x^4} + 2 = 196$$

$$x^4 + \frac{1}{x^4} = 196 - 2$$

$$x^4 + \frac{1}{x^4} = 194$$

12.If $x - \frac{1}{x} = 5$, find the values of

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

Solution:

(i) $x - \frac{1}{x} = 5$

Squaring both sides

$\left(x-\frac{1}{x}\right)^2=(5)^2$

$(x)^2+\left(\frac{1}{x}\right)^2-2(x)\left(\frac{1}{x}\right)=25 \qquad [a^2-2ab+b^2=(a-b)^2]$

$x^2+\frac{1}{x^2}-2=25$

$x^2+\frac{1}{x^2}=25+2$

$x^2+\frac{1}{x^2}=27$

(ii) Squaring both sides

$\left(x^2+\frac{1}{x^2}\right)^2=(27)^2$

$(x^2)^2+\left(\frac{1}{x^2}\right)^2+2(x^2)\left(\frac{1}{x^2}\right)=729 \quad [a^2+2ab+b^2=(a+b)^2]$

$x^4+\frac{1}{x^4}+2=729$

$x^4+\frac{1}{x^4}=727$

13.Find the continued product:

(i) $(x+1)(x-1)(x^2+1)$

Solution:

$(x+1)(x-1)(x^2+1)$

$=[(x)^2-(1)^2](x^2+1) \qquad [(a+b)(a-b)=a^2-b^2]$

$=(x^2-1)(x^2+1)$

$=(x^2)^2-(1)^2 \qquad [(a+b)(a-b)=a^2-b^2]$

$=x^4-1$

(ii) $(x-3)(x+3)(x^2+9)$

Solution:

$(x-3)(x+3)(x^2+9)$

$=[(x)^2-(3)^2](x^2+9) \qquad [(a+b)(a-b)=a^2-b^2]$

$=(x^2-9)(x^2+9)$

$=(x^2)^2-(9)^2 \qquad [(a+b)(a-b)=a^2-b^2]$

$=x^4-81$

(iii) $(3x-2y)(3x+2y)(9x^2+4y^2)$

Solution:

$(3x-2y)(3x+2y)(9x^2+4y^2)$

$= [(3x)^2-(2y)^2](9x^2+4y^2)$ $\quad [(a+b)(a-b)=a^2-b^2]$

$= (9x^2-4y^2)(9x^2+4y^2)$

$= (9x^2)^2-(4y^2)^2$ $\quad [(a+b)(a-b)=a^2-b^2]$

$= 81x^4-16y^4$

(iv) $(2p+3)(2p-3)(4p^2+9)$

Solution:

$(2p+3)(2p-3)(4p^2+9)$

$= [(2p)^2-(3)^2](4p^2+9)$ $\quad [(a+b)(a-b)=a^2-b^2]$

$= (4p^2-9)(4p^2+9)$

$= (4p^2)^2-(9)^2$ $\quad [(a+b)(a-b)=a^2-b^2]$

$= 16p^4-81$

14.If $x+y=12$ and $xy=14$, find the value of x^2+y^2

Solution:

$x+y=12$

Squaring both sides

$(x+y)^2=(12)^2$

$x^2+y^2+2xy=144$

$x^2+y^2+2\times 14=144$ $\quad (xy=14\ given)$

$x^2+y^2+28=144$

$x^2+y^2=144-28$

$x^2+y^2=116$

15.If $x-y=7$ and $xy=9$, find the value of x^2+y^2

Solution:

$x-y=7$

Squaring both sides

$(x-y)^2=7^2$

$x^2+y^2-2xy=49$

$x^2+y^2-2\times 9=49$ $\quad (xy=9\ \ given)$

$x^2+y^2-18=49$

$x^2+y^2=49+18$

$x^2+y^2=67$